U.S. Department
of Transportation
**Federal Aviation
Administration**

DRAFT Advisory Circular

Subject: AIRPLANE FUEL EFFICIENCY CERTIFICATION

Date: xx/xx/2022

AC No: 38-1

Initiated By: AEE-300

This advisory circular (AC) contains information and provides guidance on the implementation of the fuel efficiency certification requirements for airplanes that are required by Title 14 of the Code of Federal Regulations (14 CFR) part 38.

If you have suggestions for improving this AC, you may use the Advisory Circular Feedback form at the end of this AC in Appendix 6.

/Original signed by --/

Name
LOB

Table of Contents

1. Purpose

This AC provides guidance on the implementation of fuel efficiency certification requirements for airplanes. These guidelines provide an acceptable means of demonstrating compliance with the regulations of Title 14 of the Code of Federal Regulations (14 CFR) part 38. The methods and procedures described herein have been proposed as recommended certification practice for showing compliance with the fuel efficiency requirements.

See Appendix 2 for a list of acronyms, abbreviations, and symbols used in this AC.

1.1 Contents

This Advisory Circular (AC) contains information describing the standards and requirements for new aircraft fuel efficiency regulations as finalized in 14 CFR part 38 in [insert effective date of part 38]. The information contained in this document sets forth acceptable means, but not the sole means, by which compliance may be shown with the requirements of 14 CFR part 38 (part 38).

In this AC, the word "paragraph" is used to refer to a part of the AC. The word "section" (or the symbol "§") is used to refer to a part of a regulation or statute.

Pursuant to Sections 231 and 232 of the Clean Air Act Amendments of 1970 (42 USC §§7571-7572), FAA's part 38 fuel efficiency regulations implement the standards adopted by U.S. Environmental Protection Agency (EPA) in 40 CFR part 1030 (see paragraphs 4.1 - 4.3 of this AC). Any deviations from the procedures required under those regulations must be coordinated and approved by the Administrators of both the EPA and FAA (see paragraphs 4.2 and 4.3 of this AC). Users of this AC, and those subject to the regulations of part 38, are responsible for cognizance of any later amendments to 40 CFR part 1030 that may not be reflected immediately in 14 CFR Part 38 or this AC. In such instances, applicants may contact the AEE-300 Emissions Division Manager with the FAA Office of Environment and Energy at 202-267-3566 for further guidance.

1.2 Advisory Nature

The contents of this AC do not have the force and effect of law and are not meant to bind the public in any way. This AC is intended only to provide information to the public regarding compliance with existing requirements under the law or agency policies. The use of mandatory language, such as "must" and "require" in this AC describe a mandatory requirement established by statute or regulation that exists independent of the material in this AC. The term "should" is used in this AC to indicate a recommendation described in this document and not a requirement. This AC describes one or more acceptable means, but not the only means, of complying with part 38 fuel efficiency certification standards. If an applicant chooses to use the guidance presented here as an acceptable means of compliance, it must be followed in all respects to be considered a valid means of compliance.

1.3 Audience

The intended audience for this AC includes airplane performance engineers, engine specialists, and propulsion engineers from the FAA's Aircraft Certification Service (AIR) and Aircraft Certification Offices (ACOs), aircraft and aircraft engine industry specialists, aircraft certification applicants, Designated Engineering Representatives (DERs), Organization Designation Authorization (ODA) Unit Members (UMs), other certification authorities, airlines, airport operators, engineering and scientific communities, and other persons that are interested in the certification of airplanes to the fuel efficiency standards of part 38.

1.4 Intent

The intent of this AC is to provide guidance for complying with the fuel efficiency requirements of part 38.

1.5 Approval of Equivalent Procedures

Applicants may submit equivalent testing, measurement, and data reduction procedures in order to comply with part 38 airplane fuel efficiency regulations that apply to their airplane. Any alternative must be approved by the FAA's Office of Environment and Energy (AEE), at FAA Headquarters in Washington DC. Further information on equivalent procedures is provided in paragraph 6.4 of this AC. If you have further questions, please contact the AEE-300 Emissions Division Manager with the Office of Environment and Energy at 202-267-3566.

1.6 FAA Certification Process

As a part of its certification responsibilities, the FAA may verify and validate airplane fuel efficiency certification methods and data[1], or the agency may, if appropriate, delegate that responsibility to the aircraft manufacturers through a designee[2] or Organizational Designation Authorization (ODA). Agency employees may also consult FAA Order 8110.4C[3], Type Certification, and the FAA and Industry Guide to Product Certification, Third edition, May 2017[4] for more information on the agency's certification processes and procedures. Questions regarding compliance, or to request a deviation from the requirements of part 38 should be directed to AEE. Please contact the AEE-300 Emissions Division Manager in the Office of Environment and Energy at 202-267-3566.

1.7 Related Documents

Part 38 contains the fuel efficient certification requirements for certain airplanes. These regulations reference or are related to the following::

- Clean Air Act (CAA) §§ 231-232 (Aircraft Emissions Standards), 42 USC §§ 7571-7574.
- 40 CFR part 1030, *Control of Greenhouse Gas Emissions from Engines Installed on Airplanes*

Part 38 requirements are also referenced in the following regulations in 14 CFR:

- Part 21, *Certification Procedures for Products and Articles*
  - Sections 21.5, 21.17, 21.29, 21.93, 21.101, 21.115, 21.183, and 21.187.
- Part 121, *Operating Requirements: Domestic, Flag, and Supplemental Operations*
  - Section 121.141.
- Part 125, *Certification and Operations: Airplanes Having a Seating Capacity of 20 or More Passengers or a Maximum Payload Capacity of 6,000 Pounds or More; and Rules Governing Persons on Board such Aircraft.*
  - Section 125.75.

1.8 Definitions and Abbreviations/Acronyms/Symbols

Definitions, abbreviations, acronyms and symbols used in this AC that are not listed in §38.3 are included in Appendices 1 and 2 to this AC.

2. References

This AC references the following:

(1) Clean Air Act Amendments of 1970 (CAA), 42 U.S.C. § 7571 *et seq.*

(2) Control of Greenhouse Gas Emissions from Engines Installed on Airplanes; Emission Standards and Test Procedures, 40 CFR part 1030.

(3) International Civil Aviation Organization (ICAO) International Standards and Recommended Practices, Annex 16 — Volume III — Aeroplane CO_2 Emissions, May 2019

[1] 14 CFR part 38: Subpart B
[2] See 14 CFR part 183, Representatives of the Administrator
[3] https://www.faa.gov/regulations_policies/orders_notices/index.cfm/go/document.information_/documentid/15172_
[4] https://www.faa.gov/aircraft/air_cert/design_approvals/media/CPI_guide.pdf

(4) International Civil Aviation Organization (ICAO) Environmental Technical Manual (Doc 9501), Volume III — Procedures for the CO_2 Emissions Certification of Aeroplanes, May 2019

3. [Reserved]

4. Development of Airplane Fuel Efficiency Regulations

The United States is one of 192 signatory states to International Civil Aviation Organization's (ICAO) Convention on International Civil Aviation (the Chicago Convention). In March 2017, the ICAO Council adopted a new CO_2 Emissions Certification Standard with an effective date of July 2017. The ICAO considered three categories of technology associated with the fuel efficiency of an airplane design (propulsion, aerodynamics and structures) in its adoption of a CO_2 emissions certification standard for airplanes

4.1 Establishment of Airplane Fuel Efficiency Regulations

49 USC § 40105(b)(1)(A) states that the Secretary of Transportation and FAA Administrator shall act consistently with obligations of the United States Government under international agreements. For development of aviation emissions standards, the FAA leads the U.S. delegation at ICAO meetings with assistance from the EPA. Under Article 12 of the Chicago Convention, the United States, as a contracting state, is to keep its regulations uniform, to the greatest extent possible, with those established under the Convention.

4.2 Relationship Between the EPA and the FAA

The Clean Air Act authorizes the EPA to set standards for aircraft engine emissions in the United States (42 USC §7571 et seq.), while the FAA is authorized to enforce those standards under delegation from the Secretary of Transportation (42 USC §7572). The CAA establishes the EPA's authority to promulgate ICAO's emissions standards domestically in CAA § 7571:

> (2)(A) The Administrator shall, from time to time, issue proposed emission standards applicable to the emission of any air pollutant from any class or classes of aircraft engines which in his judgment causes, or contributes to, air pollution which may reasonably be anticipated to endanger public health or welfare.
>
> (B)(i) The Administrator shall consult with the Administrator of the Federal Aviation Administration on aircraft engine emission standards.
>
> (ii) The Administrator shall not change the aircraft engine emission standards if such change would significantly increase noise and adversely affect safety.

The standards promulgated by the EPA in 40 CFR part 1030 apply to certain classes of engines used by certain civil subsonic jet and propeller-driven airplanes (40 CFR part 1030.1). In developing airplane emissions regulations, the CAA requires that the EPA consult with the FAA Administrator and ensure that changes to emission standards do not significantly increase noise and adversely affect safety.

4.3 Relationship Between 14 CFR Part 38 and 40 CFR Part 1030

The fuel efficiency standards in 14 CFR part 38 implement those adopted by the EPA in 40 CFR Part 1030. Accordingly, if the EPA takes any action to change any regulation or standard in 40 CFR Part 1030 that is also promulgated in 14 CFR part 38, the Administrator of the FAA will grant a general administrative waiver of those provisions until 14 CFR part 38 can be amended to reflect the changes in 40 CFR Part 1030. The FAA will enforce the Part 1030 standards as the 14 CFR part 38 fuel efficiency regulations during any relevant airplane type certification for which a waiver has been granted. When an airplane is type certificated, the documentation will reflect that it complies with both 40 CFR part 1030 and 14 CFR part 38.

4.4 Differences Between ICAO Annex 16 Volume III and 14 CFR Part 38

Under the Chicago Convention, the FAA is obligated to notify ICAO of any U.S. regulatory differences between the ICAO standards and those adopted in the United States. Although the EPA refers to its standards as greenhouse gas emissions in part 1030, and the FAA refers to the standards and implementation requirements as fuel efficiency certification (or metric) in part 38, there are no substantive differences with the meaning of these terms and no difference between U.S. regulations and Annex 16 Volume III have been filed.

5. Part 38 Subpart A - General

5.1 Section 38.1 Applicability

Section 38.1(a) requires fuel efficiency certification for certain subsonic jets, propeller-driven airplanes, and modified airplanes. Each airplane is also subject to the requirements of 40 CFR part 1030. Fuel efficiency certification applicability is based on the date for which an application for a type certificate is submitted. In general, the standards apply to airplanes with new type designs (submitted after January 11, 2021); airplanes that were type certificated before January 11, 2021 but continue production with certain modifications after January 1, 2023; and all airplanes that are in production on and after January 1, 2028. The exact terms of applicability should be determined using the language of §38.1, not from this brief description.

For subsonic jet and propeller-driven airplanes with new type designs, part 38 applies when the criteria shown in Table 5.1 are met. *See* §§ 38.1(a)(1)-(3).

Table 5.1. Fuel Efficiency Certification Applicability for New Airplane Designs

For an airplane that is a:	With a passenger seating capacity of:	With a maximum takeoff mass (MTOM) in kg of:	For which an application for type certification is submitted:
Subsonic Jet	20 or more	> 5,700	On or after January 11, 2021
Subsonic Jet	19 or fewer	> 60,000	On or after January 1, 2021
Subsonic Jet	19 or fewer	> 5,700 or < 60,000	On or after January 1, 2023
Propeller-driven	---	> 8,618	On or after January 11, 2021

Table 5.2 shows part 38 applicability to modified versions of airplanes whose original type certificated version was not required to have fuel efficiency certification under part 38. *See* §§ 38.1(a)(4)-(5)

Table 5.2. Fuel Efficiency Certification Applicability for Modified Versions of Airplane Designs that were not required to be certified for fuel efficiency

For an airplane that is a:	With a maximum takeoff mass (MTOM) in kg of:	For which an application for type certification is submitted:
Subsonic Jet	> 5,700	On or after January 1, 2023
Propeller-driven	> 8,618	On or after January 1, 2023

Table 5.3 shows part 38 applicability to the production of individual airplanes for which the airplane type design was not originally required to meet 14 CFR part 38. *See* §§ 38.1(a)(6)-(7).

Table 5.3. Fuel Efficiency Certification Applicability for Airplanes issued Airworthiness Certificates on or after January 1, 2028

For an airplane that is a:	With a maximum takeoff mass (MTOM) in kg of:	For which the original certificate of airworthiness is issued:
Subsonic Jet	> 5,700	On or after January 1, 2028
Propeller-driven	> 8,618	On or after January 1, 2028

5.1.1 Modified Airplanes

Section 38.1(b) states that part 38 fuel efficiency requirements apply to an airplane type design that incorporates a modification, and meets the criteria in section 38.19, that changes the fuel efficiency metric value of the prior version of that airplane type design. A new airplane produced with modifications meeting the change criteria may not exceed the applicable fuel efficiency limit of part 38. The applicability criteria for modified airplanes is described in § 38.19 and guidance is provided in paragraph 6.8 of this AC. A modified airplane may not exceed the fuel efficiency metric value limit in § 38.17 that was demonstrated for the prior version of the airplane.

Applicants may choose to apply for a new fuel efficiency metric certification value even when not required by part 38.

5.1.2 Non-Applicability of the Fuel Efficiency Metric

Part 38 fuel efficiency certification requirements do not apply to the airplanes listed in § 38.1(c). These include subsonic jet airplanes having a maximum takeoff mass (MTOM) at or below 5,700 kg (§a 38.1(c)(1)) and propeller-driven airplanes having a MTOM at or below 8,618 kg (§ 38.1(c)(2)). As stated in § 38.1(c)(3)-(7),the fuel efficiency requirements of part 38 also do not apply to: :

(1) Amphibious airplanes;

(2) Airplanes initially designed, or modified and used, for specialized operational requirements;

(3) Airplanes designed with a reference geometric factor (RGF) of zero;

(4) Airplanes designed for, or modified and used for, fire-fighting purposes; and

(5) Airplanes powered by piston engines.

Airplanes in these five categories are generally designed or modified in such a way that their designs are significantly different from the typical passenger or cargo-carrying airplanes. Approval by AEE is required to determine whether an airplane fits into one of these non-applicable categories, as specified in § 38.1(c)(4). Examples of specialized operational requirements include:

(1) Airplanes that are initially certified as civil airplanes during the production process but are immediately converted to military airplanes;

(2) Airplanes with mission requirements for carrying cargo that is not possible by using less specialized airplanes (e.g., ramped, with back cargo door);

(3) Airplanes with mission requirements for conducting very short or vertical takeoffs and landings; or

(4) Airplanes with mission requirements for conducting scientific, research or humanitarian missions exclusive of commercial service.

5.2 Certification of Lower Takeoff Mass Variant

The reference masses used in the calculation of specific air range (SAR) are based on the MTOM, which is defined in §38.3 as the maximum allowable takeoff mass for the type design of an airplane. For a specific type design, applicants may develop multiple take-off mass (TOM) variants for operational purposes. The highest TOM of an airframe/engine combination must demonstrate compliance with part 38. All lesser TOMs are considered to have the same fuel efficiency certification value. In other words, certification at MTOM also certifies all TOM variants. These TOM variants would have the same certified fuel efficiency certification level as at MTOM.

Certification applicants may apply for separate fuel efficiency metric values for TOMs lower than MTOM. For a specific type design, the calculated reference airplane mass required by §38.13(b) and the maximum permitted fuel efficiency certification limits shown in §38.17 would be based on the additional TOM requested to be certified (certification at MTOM would still be required). The FEM value determined for such a TOM lower than MTOM can be also used for any TOM variants that are even lower for that specific airplane type design.

If an applicant chooses to certify a lower TOM variant, the applicant should be aware that by design of the FEM the highest weight variant (MTOM) of an airplane has the lowest margin to the regulatory limit level. Certification at MTOM is required regardless of the certification of any lower TOM variants. The 1/SAR value used in the FEM determination is calculated as an average of three reference masses (high, medium, and low).

In establishing the reference specifications for specific air range (SAR) determination, the highest SAR value usually will be sought at the maximum range cruise condition at the optimum reference specifications. A greater non-linearity in the 1/SAR versus mass relationship could be introduced by a constraint unrelated to the aerodynamic and propulsive efficiency of the airplane (e.g., an altitude pressurization limitation).

5.3 Exemptions

Pursuant to 49 U.S.C. § 44701(f), the Administrator may grant an exemption from a regulatory requirement if the Administrator finds the exemption is in the public interest. An application for an exemption from any requirements of part 38 must be submitted to the Administrator in accordance with part 11. For all part 38 exemption requests, denials, or grants, AEE will be the responsible FAA office and will consult with EPA's Designated Program Officer and other FAA offices as needed before taking any action. Petitioners for part 38 exemptions are encouraged to contact AEE prior to filing exemption requests.

5.4 Incorporation by Reference

Section 38.7 lists the documents incorporated by reference into part 38.

The ICAO documents are available for inspection at:

(1) The U,S. Department of Transportation, Docket Operations, West Building Ground Floor, Room W12-140, 1200 New Jersey Avenue, SE., Washington, DC 20590.

(2) Each FAA regional offices in Seattle, Boston, and Kansas City,

(3) The National Archives and Records Administration (NARA), For information on the availability of this information at NARA, call 202-741-6030 or go to http://www.archives.gov/federal_register/code_of_federal_regulations/ibr_locations.html

ICAO documents are available for purchase from International Civil Aviation Organization (ICAO), Document Sales Unit, 999 Robert-Bourassa Boulevard, Montreal, Quebec, H3C 5H7, Canada. http://www.icao.int/publications/Pages/default.aspx. Appendices 1 and 2 of ICAO Annex 16 Volume III are part of the following document:

International Standards and Recommended Practices, Annex 16 to the Convention on International Civil Aviation, Environmental Protection, Volume III – Aeroplane CO_2 Emissions, First Edition, July 2017, Applicable 1 January 2018.

5.5 Fuel Efficiency Compliance Demonstration Plans

Prior to undertaking a demonstration of fuel efficiency, the applicant must submit a compliance demonstration plan to the FAA. This plan would contain a complete description of the methodology and procedures by which an applicant proposes to demonstrate compliance with part 38. Compliance demonstration plans must be approved by an FAA certification office in accordance with §38.21. Use of any equivalent procedures or technical procedures not otherwise specified in part 38 must be approved by AEE and documented in compliance demonstration plans as stated in §38.21. The compliance demonstration plans would include the following information:

i. A description of the applicable sections of part 38 for the airplane

ii. Airplane description: Type, model number, and if applicable, the specific design being certificated. The applicant should demonstrate and document the conformity of the test airplane, particularly with regard to those parts which might affect the test airplane's fuel efficiency metric (FEM).

iii. FEM certification methodology which should include the means of compliance and proposed equivalent procedures.

iv. Test plan that would include:

   a. Test description. Test methods to comply with the test stability conditions in Appendix A § A38.4.2.2.2 of part 38;

   b. Test points. The planned number of test points and the planned test conditions in terms of weight or mass, center of gravity position, altitude, and speed;

c. Airplane mass. The procedures for determining the mass of the airplane is in Appendix A § A38.4.2.3 of part 38; and

d. Test fuel properties. The procedures for determining the test fuel properties is in Appendix A § A38.4.2.1.3 and A38.4.2.1.4 of part 38.

v. Deliverables: List the documents that should show compliance with Appendix A § 38.6 of part 38.

5.6 Fuel Efficiency Compliance Demonstration for Airplanes with Intermixed Engines

Typically, an applicant demonstrates compliance with part 38 for an airplane type design in which all engines are of the same design. If there are engines of different design on an airplane, the design is commonly referred to as having an “engine intermix” configuration, An applicant may choose to demonstrate compliance of an airplane type design that includes an engine intermix.

In such a case, the applicant may, subject to the approval of the FAA, demonstrate compliance in one of three ways:

a. In accordance with the test procedures defined in part 38, the test airplane must be representative of the intermix arrangement for which certification is requested; or

b. In cases where the FEM value has been established for airplanes on which each of the intermix engine models has been exclusively installed, compliance can be demonstrated on the basis of either:

i. The average of the fuel efficiency evaluation metrics for identical airplanes on which each of the intermix engine models has been exclusively installed; or

ii. The highest fuel efficiency evaluation metric for identical airplanes on which each of the intermix engine models has been exclusively installed.

In the case of time-limited engine changes, an applicant may not be required to demonstrate compliance with part 38. However, the final approval of such changes and the associated compliance demonstrations must be done by the FAA.

6. Expanded Discussion of 14 CFR part 38 Subpart B Requirements

6.1 Fuel Efficiency Metric

The fuel efficiency metric (FEM) in § 38.11 is a measure of airplane fuel efficiency. The FEM system is intended to promote advances in technologies that contribute to improvements in fuel efficiency (structural, propulsive, and aerodynamic efficiencies) as well as differentiate among generations of such technologies. In addition, the fuel efficiency metric system was designed to accommodate a wide range of technologies and designs that manufacturers may employ to improve the fuel efficiency of an airplane. The FEM system is based on three elements associated with airplane technology and design:

1. Cruise point fuel burn performance;
2. Airplane size; and
3. Airplane mass.

Section 38.11 requires that for each airplane subject to part 38, a fuel efficiency metric value is calculated using the following equation (rounded to three decimal places):

$$Fuel\ Efficiency\ \mathrm{metric}\ value\ =\frac{\left(\frac{1}{SAR}\right)_{avg}}{RGF^{0.24}}$$

The specific air range (SAR) is determined in accordance with § 38.13.The reference geometric factor (RGF) is determined in accordance with § 38.15. SAR is expressed in units of km traveled per kg of fuel burned, while RGF is dimensionless. The FEM value uses the inverse of SAR, $(1/SAR)_{avg}$, so FEM is expressed in units of kg of fuel burned per km of flight. The fuel efficiency metric is determined using an average of three SAR test points that are normalized by RGF, which is a geometric factor representing the physical size of an airplane. A lower FEM value represents greater fuel efficiency. An overview of the fuel efficiency metric system elements is shown in the schematic diagram below.

Unlike the specific fuel consumption (SFC) of an engine, which provides the engine's fuel consumption per unit of thrust (i.e., fuel efficiency of thrust production by an individual engine), SAR is a measure of fuel efficiency at the airplane level. Specifically, SAR provides the distance an airplane can travel per unit of fuel (i.e., fuel efficiency of an airplane in level cruise flight). Airplane fuel efficiency defined by SAR is commonly used by airplane manufacturers when describing airplane fuel efficiency performance to customers (e.g., airlines).

Fig. 6.1. Overview of the fuel efficiency metric system

$\underbrace{\qquad\qquad}_{}$ **Metric**		**Correlating Parameter**
(1/SAR)/RGF$^{0.24}$		**MTOM**
Specific Air Range (SAR) SAR is the distance an airplane travels in the cruise flight phase per unit of fuel consumed.	**Reference Geometric Factor (RGF)** RGF is an adjustment factor based on a measurement of airplane fuselage size derived from two-dimensional projection of the fuselage.	**Airplane Maximum Take-off Mass (MTOM)** MTOM is the highest of all take-off masses for the type design

6.2 Specific Air Range

As discussed in paragraph 6.1, Specific Air Range (SAR) is a measure of airplane cruise performance that provides the distance an airplane can travel on a unit of fuel.

In accordance with § 38.13(a), for each airplane subject to part 38, the SAR value is calculated by using either direct flight test measurements or using a performance model that is (i) validated by actual SAR flight test data; and (ii) approved by the FAA before any SAR calculations are made.

The value of 1/SAR varies with airplane gross mass and is measured at three certification test points based on gross mass. Section 38.13(b) provides that for each airplane model, a 1/SAR value is determined at each of the following reference airplane masses:

(1) High Gross Mass: 92 percent MTOM.

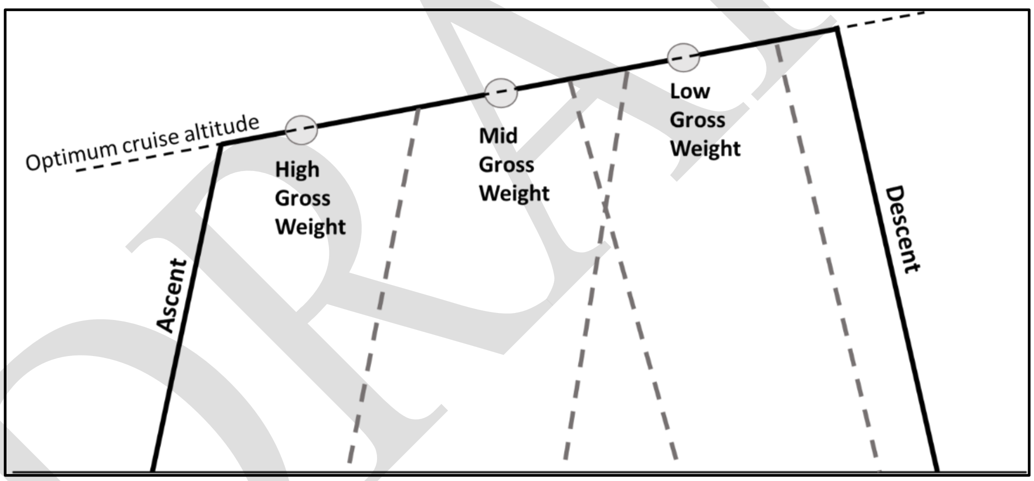
(2) Low Gross Mass: (0.45 * MTOM) + (0.63 * (MTOM^0.924)).

(3) Mid Gross Mass: simple arithmetic average of high gross mass and low gross mass

where MTOM is expressed in kilograms (kg).

Each of these certification test points represents an aircraft cruise mass that regularly occurs in service. The three reference airplane masses were chosen with the objective of making the evaluation of fuel burn performance relevant to day-to-day aircraft operations, and to make sure that the entire envelope of cruise operations is accounted for. These three 1/SAR values, calculated at each of three certification test points in accordance with §38.13(b), are weighted equally to give a single metric value. Section 38.13(c) requires that the value for $(1/SAR)_{avg}$ be obtained from the three 1/SAR points calculated in Section 38.13(b), and subsequently used to determine the fuel efficiency metric value described in §38.11. An illustrative example of the three representative cruise points is shown in the Figure 6-2.

Fig. 6.2. An illustration of the three representative cruise points.



The SAR values for each of the three reference masses defined in § 38.13(b) are to be calculated either directly from flight measurements taken at each valid test point adjusted to reference specifications, or indirectly from a performance model that has been validated by the test points, as required by § 38.13(a). The final SAR value for each reference mass is the simple arithmetic average of all valid test points at the gross mass, as in stated in § A38.5.2.3.1. No data acquired from a valid test point shall be omitted unless approved by the FAA, as stated in Appendix A § A38.5.2.3.1.

Note.— Extrapolations consistent with accepted airworthiness practices to masses other than those tested may be allowable using a validated performance model. The performance model must be based on data covering an adequate range of lift coefficient, Mach number, and thrust specific fuel consumption. There must be no extrapolation of these lift coefficient, Mach number, and thrust specific fuel consumption . Use of any performance model is subject to AEE approval.

6.3 SAR Data Analysis

The SAR data used to demonstrate compliance must consider both the selection of flight test data gathered during each test condition used for each individual SAR point, as well as the distribution of the resulting corrected SAR points relative to the three reference masses and the reference specifications.

6.3.1 SAR Measurement System

As stated in § A38.5.1.6 of part 38 Appendix A, the SAR measurement system consists of the instruments and devices, including any associated procedures, used to acquire the parameter values that are necessary for determination of the SAR for each flight test condition. Section A38.5.1.8 prescribes an accuracy level (1.5%) that much be achieved by the measurement system for the resulting SAR values to be used without penalty. If this accuracy level is not achieved, the resulting SAR values will be penalized by the amount the measurement system error exceeds the 1.5% accuracy level.

Section A38.5.1.7 defines the accuracy level of the measurement system to be the root sum of squares (RSS) of the accuracies of the individual elements that make up the measurement system. The individual elements of the measurement system are identified in § A38.5.1.6. The RSS of the measurement system is determined from the equation, $RSS = \sqrt{A^2 + B^2 + C^2 + \cdots Z^2}$ where A, B, C, ⋯ Z, are the accuracies of each of the individual elements of the measuring system. The individual elements must meet the accuracy requirements when showing compliance with part 38.

6.3.2 Selection of Flight Test Data

There are a number of methods that may be used by different airplane manufacturers in selecting flight test data for analysis, reflecting a variety of tools and practices used within the industry. Whichever method is chosen, the flight test data chosen by an applicant within a selected time frame for an individual SAR data point must meet the stability criteria detailed in § A38.4.2.2.2 or alternative stability criteria approved by the FAA as described in § A38.4.2.2.3. Test data that do not meet these stability criteria would normally be discarded if not needed. However, if such test data appear to be valid when compared with data that meet the stability criteria, and the overall stability of the conditions is reasonably bounded, these data can be used, subject to FAA approval.

One acceptable method is to employ an algorithm that automatically selects the data that meet all the stability criteria and discards data that do not. This method could be used to select the longest possible duration SAR point that meets the required stability criteria, or to select multiple SAR points of the minimum requirement duration (one minute), provided that these points are separated by a minimum of two minutes or by an exceedance of the stability criteria, as specified in § A38.4.2.2.2. Using a defined algorithm to select data in an automated process allows repeatable and consistent application to other SAR points. This method may also yield a greater number of SAR points to be used in defining the FEM value and should represent a good statistical distribution. However, because the amount of test data included in each SAR point is maximized, the resulting SAR points could exhibit more scatter than if additional selection criteria are used.

Another method for the selection of flight test data is to more closely examine the collected flight test data and select the time frame to be used to define the SAR point by choosing the best or most stable data available and ignoring less stable data that technically still meet the stability criteria. Examples of this are presented in Figures 6-3 and 6-4.

Figure 6-3 shows that the plotted parameters stay within the tolerances allowed by the stability criteria for the duration of the test condition. (The changing altitude after the end of the condition reflects pilot input to leave steady flight and transition to the next test condition.) While all parameters are within the required tolerances as specified in § A38.4.2.2.2, fluctuations in ambient temperature and Mach are evident. Figure 6-4 shows the same data, but with a manually selected range of shorter duration where the parameters are more stable.

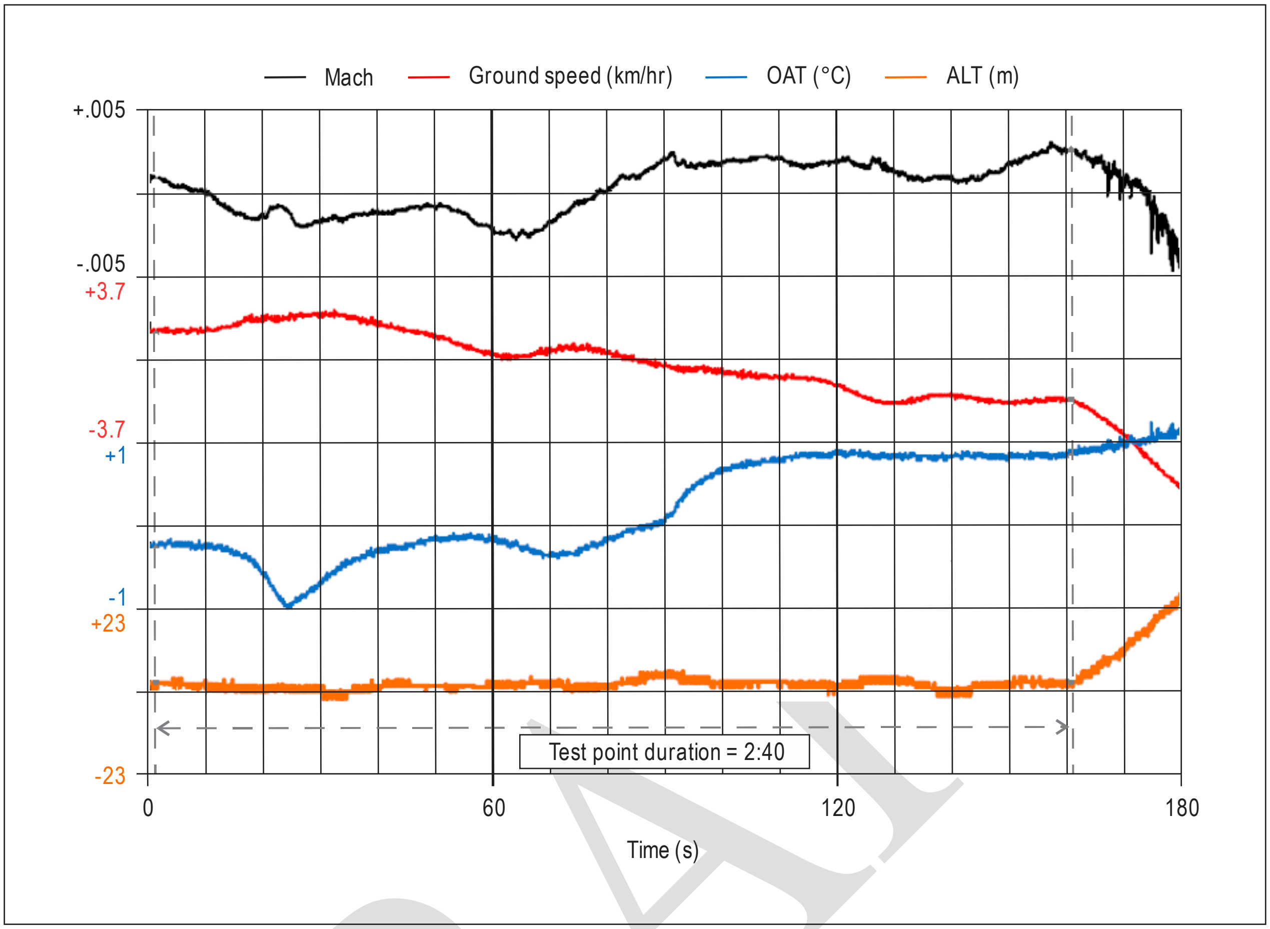


Figure 6-3. Flight test data time interval selection – Example 1

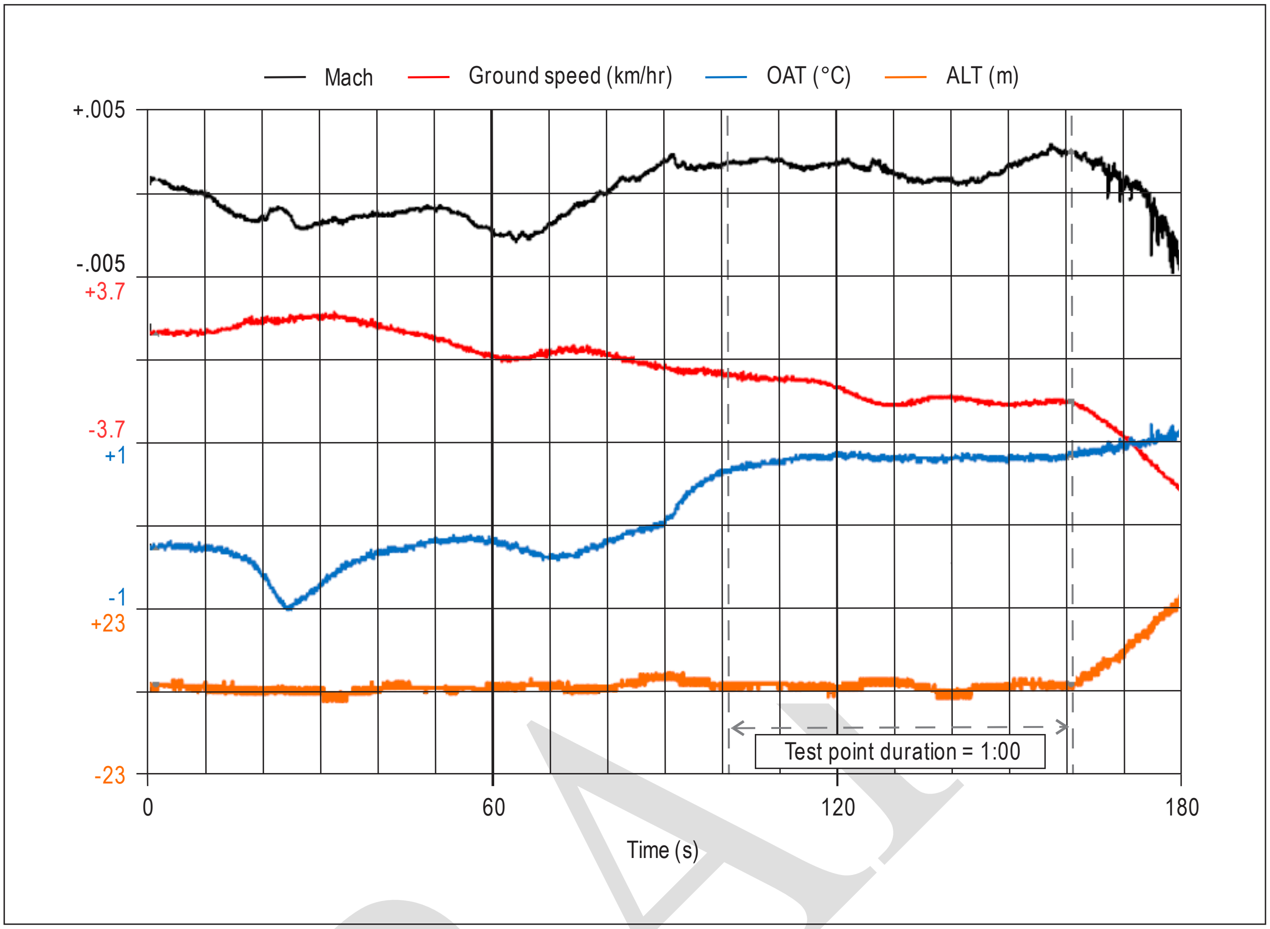


Figure 6-4. Flight test data time interval selection – Example 2

Selecting data that meet more demanding stability criteria, instead of using all data that meet the stability criteria indicated in § A38.4.2.2.2, may allow the applicant to filter out observed instabilities caused by air quality, changing environmental conditions, flight control inputs, and airplane system dynamics. This could result in a SAR point that is more representative of actual airplane performance.

Whichever approach the applicant takes to select data to define SAR points, the methodology needs to be applied as consistently as possible to minimize potential unseen bias in the resulting distribution of SAR points.

Another important aspect to consider when selecting data is to ensure that the time interval chosen is representative of the airplane's performance and not indicative of a larger trend. For example, the first plot in Figure 3-3 shows a trend line drawn through ground speed data over a 60-second time interval. This ground speed data meets the stability criteria and, taken alone, would indicate the need for an energy correction. However, if the ground speed data trace was continued over a longer time interval, it becomes apparent that it exhibits cyclical behaviour. Cyclical data need not necessarily be discarded, but the applicant should ensure that an appropriate time interval is selected such that the arithmetic average is representative. In the example shown in Figure 6-5, an energy correction would be inappropriate.

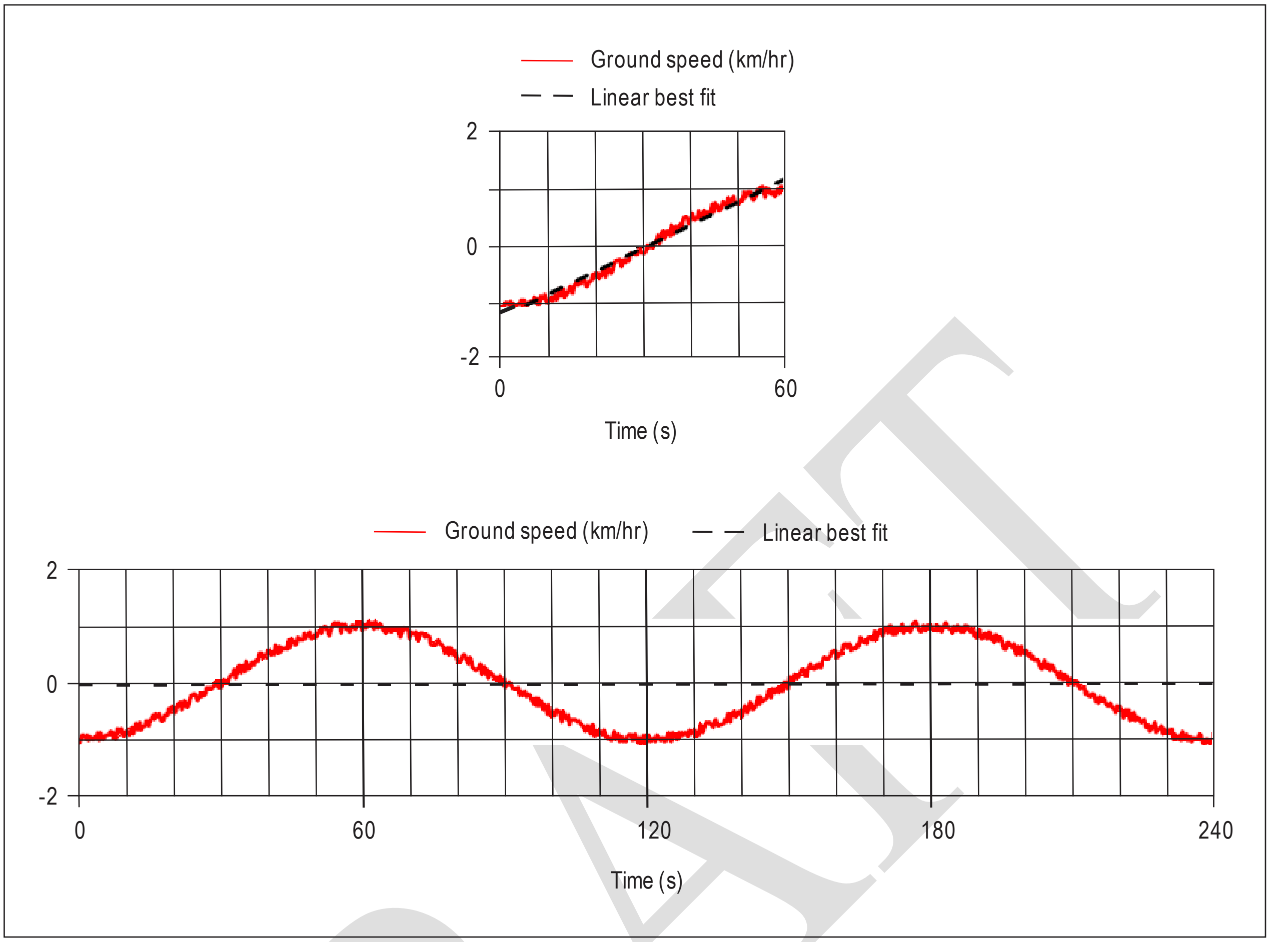


Figure 6-5. Cyclical behaviour example

6.3.3 Distribution of Resulting SAR Points

Once the individual SAR points have been selected and corrected to reference specifications, they need to be examined to ensure they reflect an accurate representation of the airplane's performance.

For example, if direct flight test is being used to collect 6 SAR points targeting one reference mass, those 6 points, when corrected to reference specifications, should result in a reasonable grouping. If 5 of the points form a reasonable grouping and one point is a clear outlier, the outlier may require closer scrutiny to ensure it is actually representative. In such a situation, collection of additional data may be warranted or, if appropriate, and subject to the approval of the FAA, the offending data point could be discarded.

If the applicant conducts tests across a range of weights to build a regression line of SAR versus weight, the collected SAR points should be reasonably distributed across the weight range. If a large portion of the regression line is unsupported by data, or is anchored by a single SAR point, then the SAR determined for one of the reference masses may be an outlier. This is an important aspect to consider during the development of the certification plan and flight test program. As with the direct test method, if a single SAR point appears to be an outlier compared to the rest of the data points, it needs to be examined more closely and may potentially be discarded.

The applicant should investigate the collection of SAR points for potential sources of unintended bias. An example would be where all of the data points collected were during periods where ground speed was increasing. If all of the test points require a large energy correction in one direction, resulting in all SAR points being significantly increased (or decreased), further scrutiny may be needed to ensure that a bias is not introduced, depending on the test data and correction techniques being used.

6.3.4 Correction of SAR Data to Reference Specifications

To ensure that the SAR data has a consistent baseline for use in determining the FEM, the SAR data must be corrected to a set of reference specifications as required by § A38.2. One set of methods for correcting the SAR data to reference specifications, but not the only acceptable methods, is provided in Appendix 3 to this AC.

6.4 Equivalent Procedures

This section provides examples of equivalent procedures. The use of these procedures must be approved by FAA in accordance with § 38.21. Section 38.13(a) establishes the methods of compliance as being direct flight test or by use of a validated performance model. For example, if an application for certification of a fuel efficiency metric value involves only a minor change to an airplane type design, the resulting change in the fuel efficiency metric value may be established using an equivalent analytic procedure approved by the FAA.

6.4.1 Approval based on existing data

Using existing data, applicants should develop a regression curve approach for SAR values across the gross weight (MTOM) range. This regression curve approach using existing data could be used in situations such as:

a) The existing model used company test data that may have not been witnessed by the FAA or documentation of the conformity of the airplane type design may not have been provided to the FAA, but the data obtained are found acceptable by the FAA at the time of certification.

b) The accuracy of the instrumentation and the data reduction processes may not have been documented sufficiently for certification, or the original documentation may not have been retained, but the data available are found acceptable by the FAA at the time of certification.

6.4.2 Approval of a change based on back-to-back testing

The use of back-to-back test data may be requested by applicants as an equivalent procedure for determining the FEM for a relatively small type design change (e.g., antenna installations or other simple drag changes). Back-to-back testing for determining a FEM value change means that a test is first completed without the change applied to the airplane, and then the test is repeated with the change made to the airplane. The difference in the FEM values of the before and after tests is presumed to be the result of the change on the airplane FEM value. This approach will typically not be appropriate for engine changes where the specific fuel consumption (SFC) of the engine may change due to internal changes. This compliance approach will likely be especially useful for supplemental type certificate (STC) modifiers who do not have access to the original flight test data from the airplane manufacturer.

c) Back-to-back testing should be performed on the same airplane and engines with the modification installed and not installed.

d) Instrumentation adequate to provide data meeting the accuracy requirements of the standard should be installed in the test airplane.

e) The data reduction and comparison processes must be approved by the FAA in accordance with § 38.21.

6.4.3 Approval of a change based on analysis

The use of analytical processes may be requested by an applicant to evaluate a change to the FEM of a previously approved airplane type design, and establish compliance with § 38.17; the analytical processes must be approved by the FAA in accordance with § 38.21. Paragraphs 6.4.3.1 and 6.4.3.2 below provide a non-exclusive list of possible analytical methods, but do not preclude the use of other equivalent procedures.

6.4.3.1 Changes affecting characteristics of airplane aerodynamics

Subject to approval by the FAA, methods available to evaluate the effect of aerodynamic changes on airplane drag could include:

a) Semi-empirical methods (SEM),

b) Computational fluid dynamics (CFD),

c) Computational fluid dynamics combined with computational structural mechanics (CFD-CSM), and

d) Wind tunnel testing.

The FAA's approval for the use of the above analytical methods to evaluate a change to the FEM value may depend on the type of airplane modification as listed below.

a) For design changes affecting parasitic drag and/or profile drag (e.g., friction drag and/or viscous pressure drag), SEM, CFD, or wind tunnel testing could be used. In this case, it is assumed that airplane aeroelastics are not affected.

b) For all other design changes, CFD could be used for items not affecting airplane aeroelastics, CFD-CSM could be used for items affecting airplane aeroelastics, and wind tunnel testing could be used in all cases.

6.4.3.2 Changes to specific fuel consumption (SFC) affected by a change to the propulsion system:

Possible methods available to evaluate the effect of propulsion system design changes on specific fuel consumption (SFC) are:

a) Thermo dynamical models (engine performance decks);

b) Computational fluid dynamics (CFD);

c) Component testing;

d) Full engine testing; and

e) A combination of a) through d).

6.4.4 Approval based on first principles models

Introduction

Section 38.13(a) requires that the specific air range must be determined by direct flight test measurements or the use of a performance model that is --

(i) validated by actual SAR flight test data and

(ii) approved by the FAA before any specific air range calculations are made.

A performance model is defined in 38.3 as "*an analytical tool (or a method) validated using corrected flight test data that can be used to determine the specific air range values for calculating the fuel efficiency metric value.*"

Guidance on the approval of SAR values based on specific types of performance models known as “first principles models” is provided below. A first principles model is defined as a tool that enables the derivation of a SAR value, in given flight conditions, from flight mechanics equations using airplane aerodynamics and engine performance data.

First principle models should be validated by actual SAR flight test data that is acquired in a manner consistent with § A38.1.2 and as described in paragraph 6.3 of this AC. Validity of the model need only to be shown for the test points and conditions relevant to demonstrating compliance with § 38.17.

The method proposed in this paragraph 6.4.4 represents one possible method to show compliance with § 38.17.

6.4.4.1 Selection of relevant flight test points for the model validation

First principles SAR models developed by airplane manufacturers typically cover most of the flight envelope to provide many different operational fuel planning requirements. The range of flight test data from these models typically spans large Mach and weight/δ ranges.

The example shown in Figure 6-6 below illustrates the wide distribution of the flight test data across the model, and that none of the test points generally fall exactly at the three SAR conditions defined in § 38.13(b).

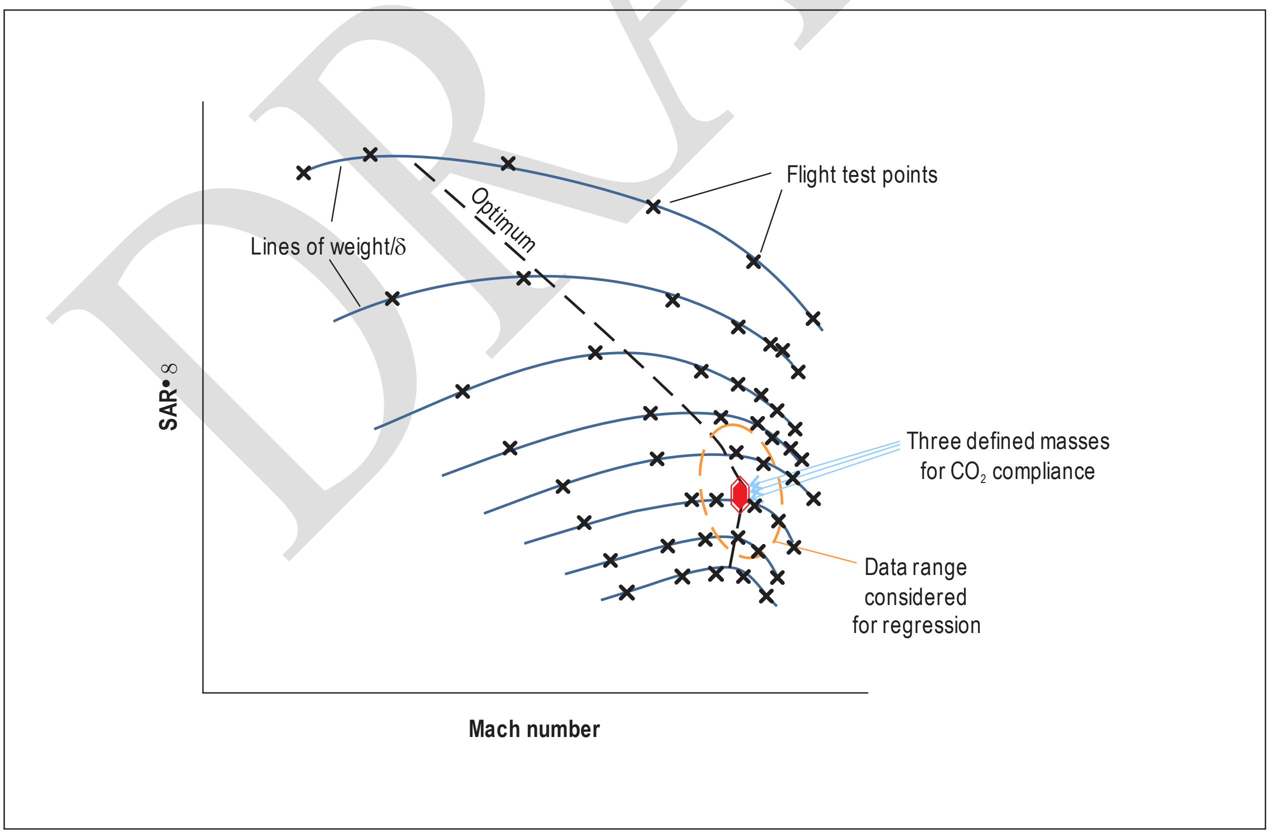


Figure 6-6. Test points range

Figure 6-6 shows that the three SAR conditions required by§ 38.13(b) are relatively close to each other in terms of Mach and weight/δ because they represent the same optimum aerodynamic conditions. Although these three SAR conditions may not have been specifically flight tested, they are generally well framed by other relevant flight test data in a local Mach and weight/δ range.

In order to validate the performance model in conditions relevant to demonstrating compliance with § 38.17, flight test points around the optimum Mach number and optimum weight/δ would be selected based on the following criteria:

a) Weight/δ between ±5 percent of the optimum weight/δ; and

b) Mach number between -0.02 and +0.015 of the optimum Mach number.

Note 1.— The ±5 percent value in a) is used to capture flight test points on weight/δ lines above and below the optimum weight/δ included in the airplane manufacturer's flight test practices.

Note 2.— The density of test points is generally lower at Mach numbers below the optimum Mach number and higher at Mach numbers above the optimum Mach number. Additionally, the Mach effect on SAR is generally less at Mach numbers below the optimum Mach number than above, and a sharp SAR decrease may occur when approaching high Mach numbers. For this reason, the Mach range for the test point selection is larger at Mach numbers below the optimum Mach number (-0.02) than above (+0.015).

Note 3.— Testing target weight/δ values non-dimensionalizes the pressure altitude so that weight/δ varies as $C_L.M^2$. Therefore, weight/δ at a given Mach varies similarly to C_L at that Mach number.

6.4.4.2 SAR model justification

6.4.4.2.1 Flight test SAR versus SAR model

In order to use of first principle models to determine the SAR values for calculating the FEM, manufacturers are expected to explain their modelling validation principles and show that their modelling correctly matches flight test data acquired in a manner consistent with part 38 Appendix A, as described in paragraph 6.3 of this AC.

As shown in Figures 6-7 and 6-8 below, one possible method is to present plots of percent ΔSAR (i.e. Measured SAR minus Computed SAR) versus Mach and versus weight/δ (or C_L). These should be shown in the Mach and weight/δ (or C_L) range of the cruise envelope.

An increased number of test points, when available, may be used to qualitatively show that the test point selection, described in paragraph 6.4.4.1 above (red points in Figures 6-7 and 6-8 below), is not differently scattered or biased than other test points in the cruise envelope (green points).

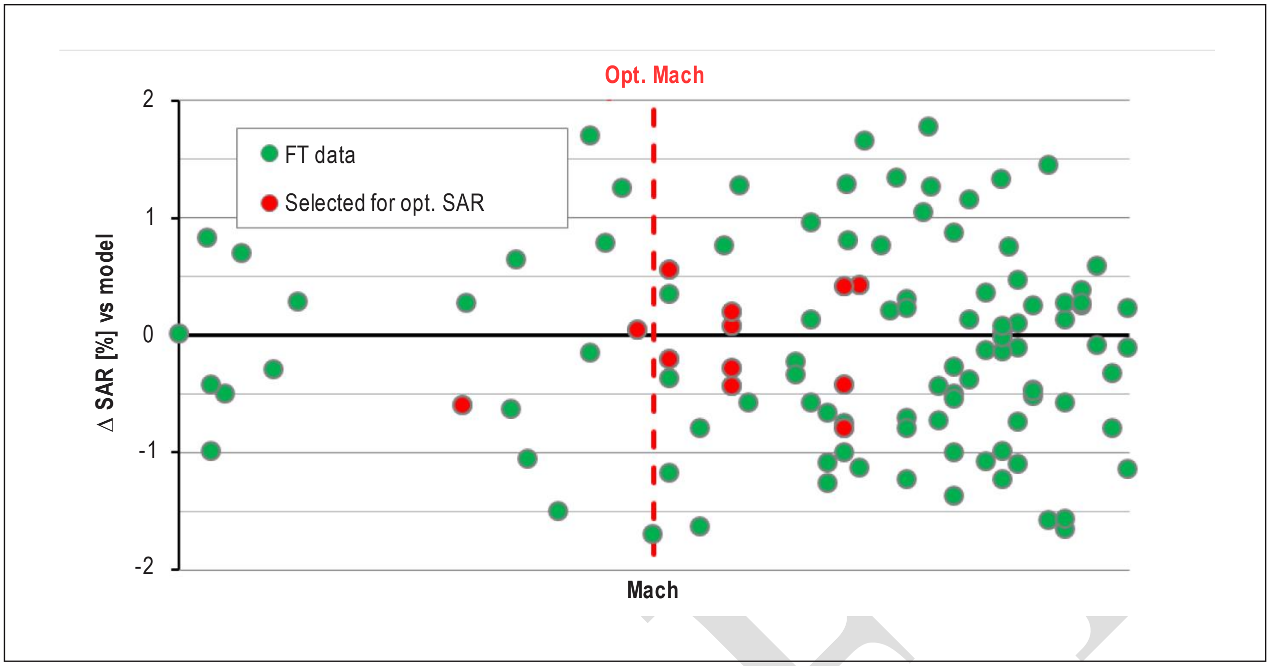


Figure 6-7. ΔSAR versus Mach number

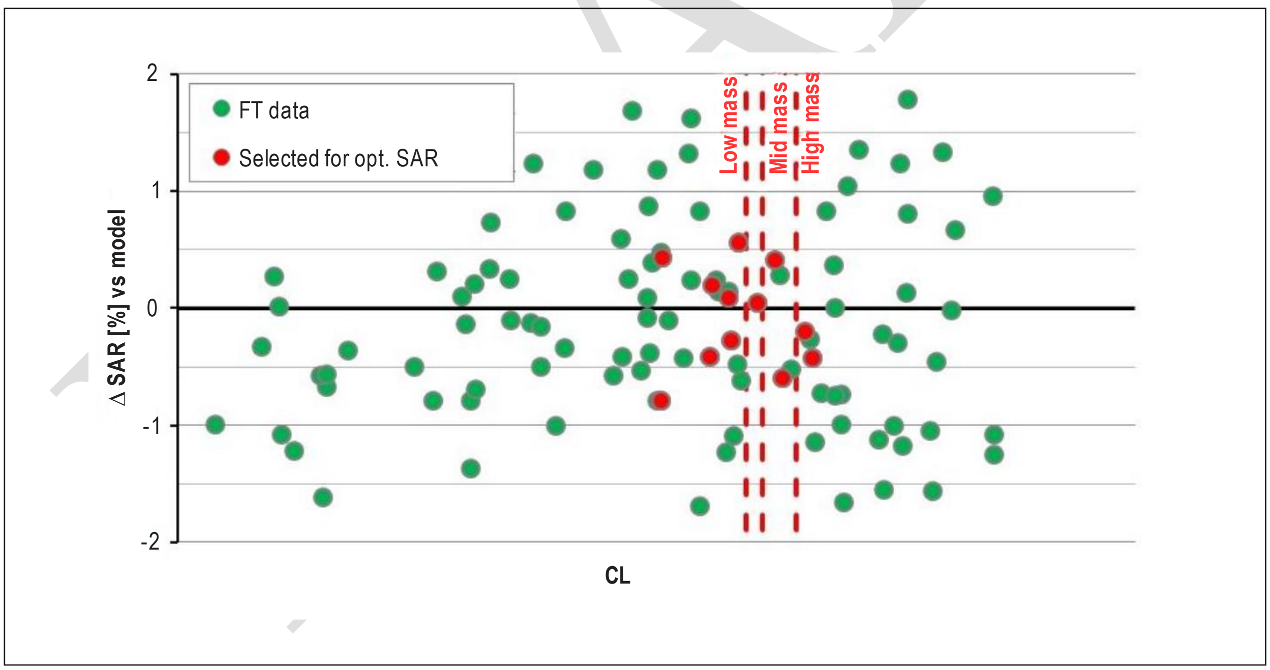


Figure 6-8. ΔSAR versus C_L

6.4.4.2.2 SAR model presentation

The comparison of flight test points with a model does not validate how that model is built and how the optimum conditions represent the FEM. Accordingly, the SAR model used to demonstrate compliance with § 38.17 must be presented to the FAA.

One possible method to present the SAR model is to present plots of computed SAR versus Mach and computed SAR versus C_L for the three masses referenced in § 38.13(b). (See Figures 6-9 and 6-10 below), and to show how the SAR model is supported by flight test data.

The objective of these plots is to show that there are no abnormal discontinuities in the Mach and C_L ranges selection around SAR conditions used for the FEM determination.

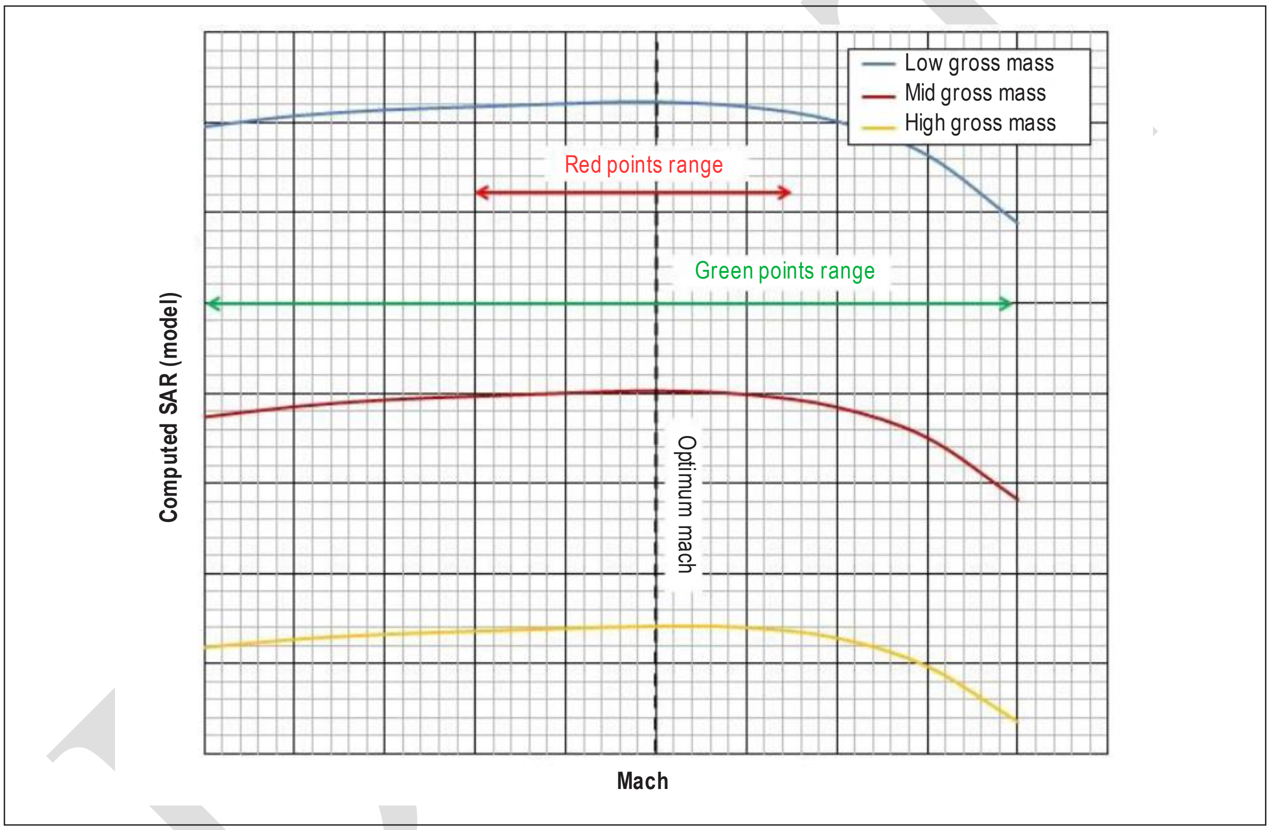


Figure 6-9. Computed SAR (model) versus Mach at optimum altitude

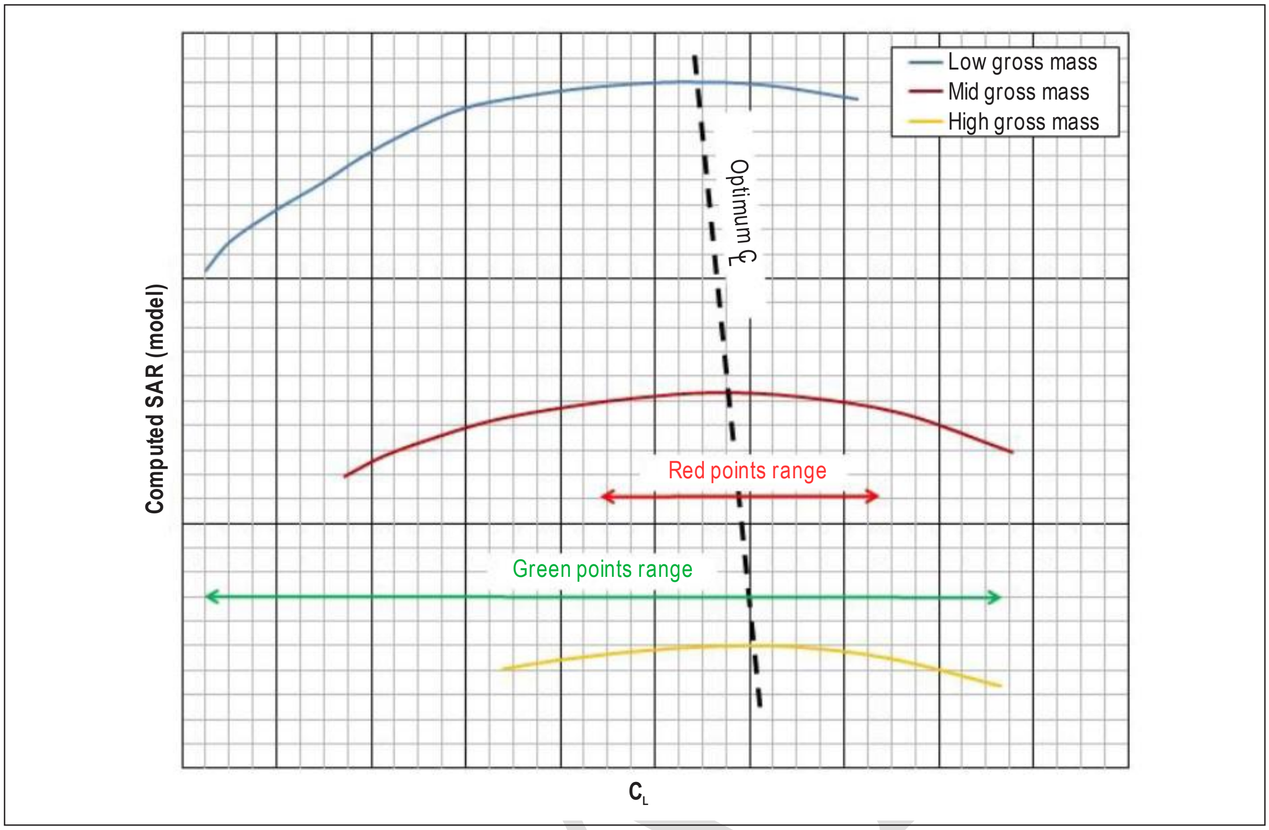


Figure 6-10. Computed SAR (model) versus C_L at optimum Mach

6.4.4.3 SAR model validation – confidence interval determination

As explained in paragraph 6.4.4 above, the SAR model needs to be validated under the conditions relevant to demonstrating compliance with § 38.17. To be used for validation, flight test points need to be selected using the criteria described in paragraph 6.4.4.1 above. The number of flight test points selected will represent n measurements of SAR (SAR_1, SAR_2, …, SAR_n) obtained in approximately the same conditions of Mach and C_L.

Step 1 — Estimate of the mean ΔSAR value and of the standard deviation

For each flight test point (i), the SAR difference $\Delta SAR_{(i)}$ between the measured SAR and the computed SAR can be determined as follows (in percent):

$$\Delta SAR_{(i)}(\%) = [(Measured\ SAR_{(i)} - Computed\ SAR_{(i)}) / Computed\ SAR_{(i)}] \times 100\ (\%).$$

The estimate of the mean ΔSAR value in percent, $\Delta SAR_{AVG}(\%)$, can then be determined for the n measurements as follows:

$$\Delta SAR_{AVG}(\%) = estimate\ of\ the\ mean\ \Delta SAR\ (\%) = \frac{1}{n}\left\{\sum_{i=1}^{i=n} \Delta SAR_{(i)}\right\}(\%).$$

The estimate of the standard deviation of the mean is found as follows:

$$s = estimate\ of\ the\ standard\ deviation\ of\ the\ mean = \sqrt{\frac{\sum_{i=1}^{i=n}\left(\Delta SAR_{(i)} - \Delta SAR_{AVG}\right)^2}{n-1}}\ (\%).$$

Step 2 — Confidence interval determination

The low number of test points in a small range of Mach and C_L results in a statistical population that follows the Student law, and allows the 90 percent confidence interval to be defined in accordance with the methodology for Direct Flight Testing, as described in appendix 4 of this AC. The objective is to determine an average ΔSAR value (%) and its associated 90 percent confidence interval. The minimum acceptable sample size is twelve points (see § A38.5.3.4.).

Following this step and using the Student t-distribution, the 90 percent confidence interval (CI_{90}) for the estimate of the mean ΔSAR_{AVG} can be determined as:

$$CI_{90}\,(\%) = \left[\Delta SAR_{AVG} \pm t_{(.95,n-1)}\frac{s}{\sqrt{n}}\right] (\%)$$

where $t_{(.95,\,n-1)}$ = Student t-distribution (for 90 percent confidence) for n degrees of freedom. (See Table A4-1 in appendix 4 of this AC).

Step 3 — Validity of results

Section A38.5.3.2 contains the following requirements,

"A38.5.3.2 If the 90 per cent confidence interval of the SAR value at any of the three reference airplane masses --

A38.5.3.2.1 Is less than or equal to ±1.5 per cent, the SAR value may be used.

A38.5.3.2.2 Exceeds ±1.5 per cent, a penalty equal to the amount that the 90 per cent confidence interval exceeds ±1.5 per cent must be applied to the SAR value, as approved by the FAA."

Applying those requirements to a use of a first principles model for showing compliance:

a) If the 90 percent confidence interval remains within the ±1.5 percent limits, then the ΔSAR_{AVG} (%) value can be retained as the reference deviation for model correction.

b) If the 90 percent confidence interval exceeds the ±1.5 percent limits, then a penalty equal to the amount that the 90 percent confidence interval exceeds ±1.5 percent should be applied to the ΔSAR_{AVG}(%) value, subject to the approval of the FAA. The corrected ΔSAR_{AVG}(%) value then becomes the reference deviation for the model correction.

Step 4 — SAR model correction

When used to determine the FEM, the SAR values computed with a first principles model at each of the three reference airplane masses must be corrected by a percentage amount that is equal to the reference deviation determined in Step 3.

6.4.4.4. Example of the determination of 90 percent confidence intervals

The set of 12 flight test points are presumed to meet the selection criteria of paragraph 6.4.4.1 above. The difference between the measured SAR and the computed SAR has been established as follows:

Table 6-1. Test point selection

Selected test point number	*ΔSAR (%)*
1	0.08
2	-0.6
3	-0.42
4	0.19
5	-0.43
6	0.23
7	-0.28
8	0.45
9	0.10
10	-0.28
11	-0.80
12	-0.64

a) The number of data points (n) = 12.

b) The degrees of freedom (n-1) = 11.

c) The Student t-distribution for 90 percent confidence and 11 degrees of freedom ($t_{(.95\,,11)}$) = 1.797. (See examples in Table A4-1 in appendix 4 to this AC.).

Step 1 — Estimate of the mean ΔSAR (%) and of the standard deviation(s)

$$\Delta SAR_{AVG}(\%) = \frac{1}{12}\left\{\sum_{i=1}^{i=12} \Delta SAR_{(i)}\right\} = -0.200\ \%$$

$$s = \sqrt{\frac{\sum_{i=1}^{i=12}\left(\Delta SAR_{(i)} - \Delta SAR_{AVG}\right)^2}{n-1}} = 0.39950\ \%$$

Step 2 — 90 percent confidence interval (CI_{90})

$$CI_{90} = \Delta SAR_{AVG} \pm t_{(.95,11)} \frac{s}{\sqrt{12}} = -0.200 \pm 0.207\ \%$$

Step 3 — Check of confidence interval limits

The confidence interval extends to ±0.207 percent around the mean ΔSAR value (-0.200 percent), which is within the confidence interval limit of ±1.5 percent. As a result, a ΔSAR value of -0.200 percent can be retained for model correction.

Step 4 — SAR Model correction

The SAR value calculated for each reference mass of the FEM can be determined by computation with the first principles model and would be corrected by the -0.200 percent value determined in step 3 as follows:

$SAR_{corrected} = SAR_{computed} \times 0.998$

6.5 Validity of Results – Confidence Interval

Section A38.5.3.1 requires the 90 percent confidence interval to be calculated for each of the SAR values at the three reference masses. Appendix 4 provides guidance regarding confidence interval evaluation, including examples.

6.6 Reference Geometric Factor

6.6.1 RGF General Guidelines

The reference geometric factor (RGF) is a non-dimensional parameter used to adjust the value of $(1/SAR)_{AVG}$. RGF is based on a measure of fuselage size normalized with respect to 1 m2, and is derived as prescribed in §38.15.

The basic concept of RGF, and the exponent of 0.24 for RGF in the fuel efficiency metric, is to give appropriate credit for the size of the cabin. The RGF represents the area that is available for the transport of passengers or cargo on the main deck, and on an upper deck. Sections 38.15 and § A38.3 provide a description of RGF and list some areas that are to be included in the RGF calculation. This list of included areas is not comprehensive; RGF also includes, for example, any space in the cabin used for wardrobes, or for cabinets containing equipment such as passenger entertainment systems or life rafts. If the interior configuration of an airplane is changed in service, even to the extent of converting a passenger airplane into a freighter, there is most likely no reason to change the RGF.

Most airplanes have a single deck, or floor, for passengers or cargo. Many single-deck airplanes also have an underfloor cargo bay, but that does not mean that they should be considered as two-deck airplanes.

6.6.2 Orthogonal Projection and Width Boundary

For a single-deck airplane, the RGF is determined by making an orthogonal projection of the outer mold line (OML) of the fuselage pressure shell, onto a plane that is parallel to the main deck floor. The term orthogonal means that the projection is made using theoretical lines of projection which are orthogonal, or perpendicular, to the main deck floor, and also orthogonal to the plane where the projection is made. In the case of an airplane in which the main deck is perfectly horizontal when it stands on the ground, the projection is similar to the ‘shadow area’ on the ground, or the top view area of the fuselage. The terms ‘shadow area’ and ‘top view’ can be misleading however, and orthogonal projection is the proper term for obtaining a true projection. The term ‘shadow area’ could imply projection lines emanating from a single point, effectively like the sun. It is possible to make a projection using lines which emanate from any single point, but that is not intended, and would not be a true projection for RGF. The term ‘top view’ can be misleading for an airplane in which the main deck is not perfectly horizontal when it stands on the ground. Typically the main deck of an airplane is at a constant height (or constant Z station) in terms of the X, Y, Z coordinate system for the airplane, and in that case the orthogonal projection is the same as a plan view drawing, viewing either up or down along the Z axis. However, there is no requirement for any deck to be at a constant Z station.

Typically the maximum width of the fuselage cross section is above the main deck for a single-deck airplane, and if the fuselage has a constant cross section for part of its length, then the maximum width will be at the same height above the main deck (i.e. at the same Z station) for part of the fuselage length. At the aft end of the fuselage however, there will no longer be a constant cross section, and it is common for the point of maximum width to rise up higher above the main deck towards the tail, with fuselage upsweep. Towards the nose, typically the point of maximum width descends closer to the main deck. For the orthogonal projection, the maximum external width of the OML is always ‘captured’ at whatever height above the deck that it occurs.

For an airplane that has a main deck and also an upper deck, the RGF for the main deck is determined using the fuselage OML as the width boundary, as for a single-deck airplane. The upper deck is considered separately, and the width boundary for the upper deck RGF is defined as the fuselage OML at or above the upper deck floor level. This means that to determine the upper deck RGF, all of the fuselage below the upper deck floor can be removed from consideration, and then an orthogonal projection should be made of the remaining part of the fuselage, the entire upper cabin, onto a plane that is parallel to the upper deck. This projection will capture the maximum width of the fuselage OML at any height at or above the upper deck floor. The upper deck may be parallel to the main lower deck, but there is no requirement for this to be the case. The total RGF is simply the sum of the RGF values for the two decks.

6.6.3 Forward and Aft Boundaries

The aft boundary to be used for calculating RGF is the aft surface of the aft pressure bulkhead skin. This bulkhead may be a flat or domed skin to take the pressure loads, and typically has supporting structure attached either to the forward or aft face of the bulkhead. This aft boundary definition is consistent with the concept of using the external surface of the pressure shell for the width boundary.

The forward boundary to be used for calculating RGF is the forward surface of the forward pressure bulkhead skin except for the cockpit crew zone. For all airplanes, a plane will need to be defined for the aft boundary of the cockpit crew zone. If there is a cockpit door, this aft boundary is the plane of the forward face of the cockpit door. If there is no cockpit door, a boundary plane will need to be defined that is aft of the cockpit crew seats. Some airplanes may have optional approved cockpit layouts, for example to provide for one or two ‘jump seats’ aft of the two standard pilot seats; in that case the RGF should be based on the smallest approved cockpit layout (see § A38.3).

Some airplanes are certified for single-pilot operations, as illustrated in Appendix A38.3 Figure A38-2. This figure shows that a passenger may be in the right cockpit seat, and the right side of the cockpit can be included in the RGF, even if a cockpit door is standard equipment. In this case, the aft boundary of the cockpit crew zone applies to the left side of the fuselage only.

For an airplane with two decks, the cockpit may be accessed from the main deck, or the upper deck, but the two decks are considered separately. One deck may extend forward, above or below a cockpit crew zone, and in that case the aft boundary of the cockpit crew zone should only apply to the RGF of the deck that provides direct access to the cockpit, not to both decks.

6.6.4 Floor Plane

For the floor plane of the main deck, and any upper deck, it is assumed that most, or all, of the floor area is on the same flat plane. It is common to have some local anomalies, such as a step up in the floor of the cabin towards the tail, typically in a baggage area. If there is a relatively small, elevated floor it is usually flat and parallel to the main floor, but there is no requirement for it to be so. Some airplanes may have other local floor changes, possibly even changes in slope. Some smaller airplanes have a dropped aisle, with elevated seating areas on both sides of the aisle. There is no requirement for any deck floor to be parallel with the upper or lower skin of the fuselage, but this will usually be the case. For each deck, there should be no difficulty in identifying a nominal flat plane which should apply to it, which then allows for the identification of a true flat plane (with no local anomalies) parallel to the deck, for the purpose of the orthogonal projection.

6.6.5 RGF Sensitivity

Although RGF is an important component of the CO_2 emissions evaluation metric value, the exponent of 0.24 means that the metric value is less sensitive to a change in RGF as compared to a change in the average 1/SAR value. As an example, if the average 1/SAR value is changed by a factor of 1.01, then the metric value will also change by a factor of 1.01 (a 1% increase). However, if the RGF changes by a factor of 0.99 (a 1% decrease) then the metric value will increase by only a factor of 1.0024150 (an increase of 0.2415%). The change in metric value as a result of a change in RGF, can be expressed as the reciprocal of the factor applied to RGF, with an exponent of 0.24 applied. For this example: (1 / 0.99) = 1.010101 ; and (1.010101)0.24 = 1.002415.

The RGF sensitivity may lead an applicant to declare positions for the forward boundary and aft boundary that are slightly conservative in order to avoid lengthy geometric explanations and justifications. Typically, a small amount of conservatism, by declaring a forward and/or aft boundary that is displaced by 1 or 2 cm will lead to a very small change in RGF, much less than 1%, and an extremely small penalty in the metric value of less than 0.1%.

An applicant may also consider the potential for future geometric changes, and the no-FEM-change criteria. It may be administratively easier to deal with a small geometric change in the future, if the RGF has been defined in a slightly conservative way.

6.7 Fuel Efficiency Limits

Section §38.17 provides the maximum permitted fuel efficiency metric values for airplanes defined under 38.1(a), as determined pursuant to 38.11. In accordance with 38.17(b), the fuel efficiency metric value may not exceed the values detailed in the chart below. Notably, the values determined in accordance with the equations in the table below should be rounded to three decimal places. The maximum permitted fuel efficiency metric values are based on the type of airplane as defined in 38.1 (applicability section) and the airplane's MTOM. A more detailed discussion on the applicability of 38.1 can be found in paragraph 5.1 of this AC.

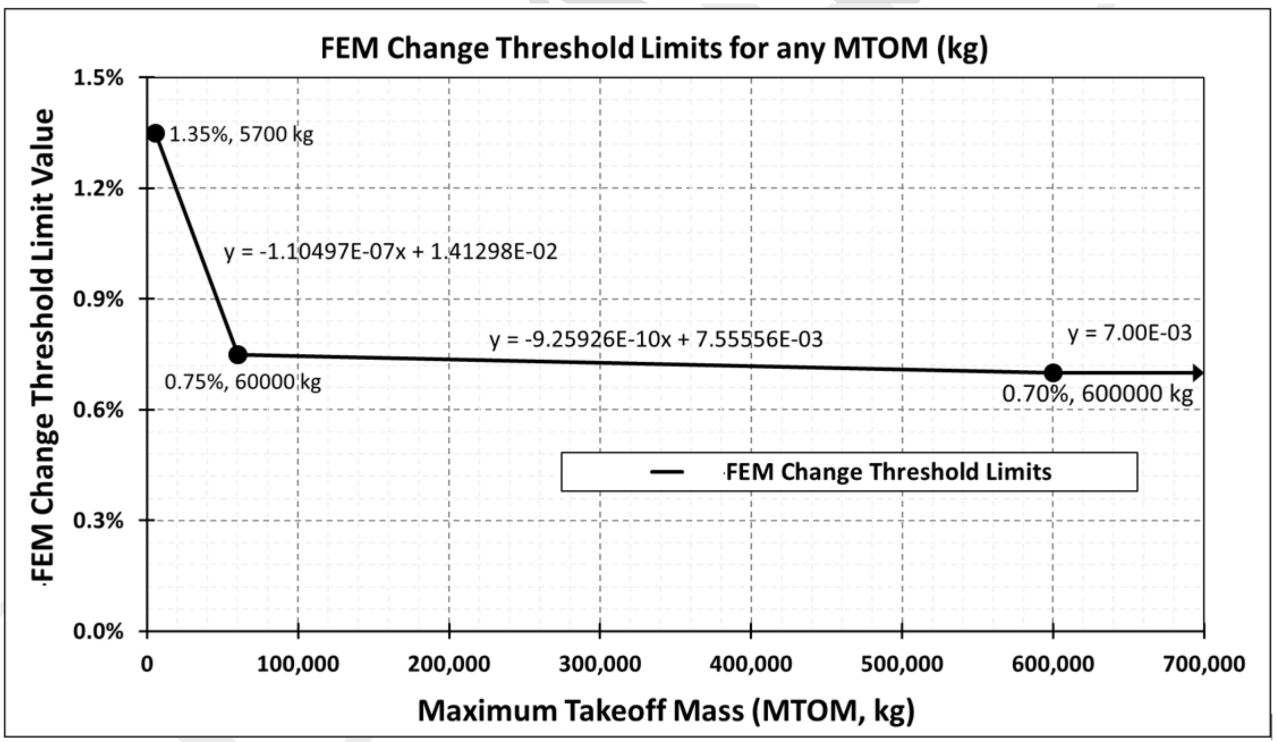
6.8 Change Criteria

Section §38.19 requires the demonstration of compliance with the fuel efficiency limits of §38.17 for airplanes with a change in design that either increases the MTOM, or results in certain specified increases in the FEM value. If the FAA determines that the modifications do not increase MTOM or exceed the fuel efficiency metric threshold limits in §38.17, the airplane is considered to be in compliance with that regulation.

Changes to an airplane that has shown to comply with §38.17 and that neither increase its MTOM nor increase its FEM value by more than the thresholds identified above are not considered to be FEM changes. Figure 6.5 may be used to visually evaluate the FEM change threshold limits of §38.19 for any MTOM.

For airplanes incorporating changes that are below the FEM change threshold limits, the FEM value of the changed type design configuration is considered the same as the parent type design.

Fig. 6.5. FEM change threshold limits with variation in MTOM



7. Additional Guidance

If an applicant needs additional guidance, contact the AEE-300 Emissions Division Manager with Office of Environment and Energy at 202-267-3566.

Appendix 1: Definitions

The following terms are defined in § 38.3, which is reproduced here for convenience:

Amphibious Airplane means an airplane that is capable of takeoff and landing on both land and water. Such an airplane uses its hull or floats attached to the landing gear for takeoff and landing on water, and either extendable or fixed landing gear for takeoff and landing on land.

ICAO Annex 16, Volume III means Volume III of Annex 16 to the Convention on International Civil Aviation (incorporated by reference in § 38.7).

Maximum takeoff mass (MTOM) is the maximum allowable takeoff mass as stated in the approved certification basis for an airplane type design. Maximum takeoff mass is expressed in kilograms.

Performance model is an analytical tool (or a method) validated using corrected flight test data that can be used to determine the specific air range values for calculating the fuel efficiency metric value.

Reference geometric factor (RGF) is a non-dimensional number derived from a two-dimensional projection of the fuselage.

Specific air range (SAR) is the distance an airplane travels per unit of fuel consumed. Specific air range is expressed in kilometers per kilogram of fuel.

Subsonic means an airplane that has not been certificated under 14 CFR to exceed Mach 1 in normal operation.

Type certificated maximum passenger seating capacity means the maximum number of passenger seats that may be installed on an airplane as listed on its type certificate data sheet, regardless of the actual number of seats installed on an individual airplane.

Appendix 2: Abbreviations/Acronyms/Symbols

The abbreviations/acronyms/symbols used in this AC have the following meanings whether expressed in upper or lower case:

a	Earth's radius at the equator
A	Airplane reference wing area
b	Earth's radius at the poles
B	Value representing the variation of drag with Reynolds number
AEE	FAA's Office of Environment and Energy
AVG	Average
C_D	Drag coefficient
CFD	Computational fluid dynamics
CG	Center of gravity
CI	Confidence interval
CI_{90}	90% Confidence interval
C_L	Lift coefficient
CO_2	Carbon dioxide
EPA	United States Environmental Protection Agency
FAA	Federal Aviation Administration, United States Department of Transportation
FEM	Fuel Efficiency Metric
g	Gravitational acceleration
g_0	Standard acceleration due to gravity at sea level and a geodetic latitude of 45.5 degrees
g_{ref}	Gravitational acceleration, based on g_0, for an airplane travelling true north in still air at the reference altitude and a geodetic latitude of 45.5 degrees
$g_{Coriolis}$	Component of gravitational acceleration that is due to Coriolis effect
g_{cent}	Component of gravitational acceleration that is due to centrifugal effect
g_{test}	Gravitational acceleration at the test values of latitude, geometric altitude, true track, and ground speed
$g_{\varphi,alt}$	Component of the gravitational acceleration at the test altitude and latitude at zero ground speed
h	Altitude
ICAO	International Civil Aviation Organization
Kg	Kilogram(s)
LHV	Lower heating value
m	Meters
M	Mach number
MTOM	Maximum Takeoff Mass
n	Number of data points
N	Newtons
ODA	Organizational Designation Authorization
OML	Outer Mould Line
Pa	Pascals
P_S	Static pressure
r_e	Earth's radius
RE	Reynolds number
RGF	Reference Geometric Factor
s	Time in seconds or estimate of the standard deviation of the mean
SAR	Specific Air Range
SEM	Semi-empirical methods
SFC	Specific fuel consumption
STC	Supplementary type certificate
T	Temperature
$t_{(.95, n-1}$	Student t-distribution for 90 percent confidence for n degrees of freedom
TAS	True airspeed
TC	Type certificate
TOM	Takeoff mass
TSFC	Thrust Specific Fuel Consumption

V	Speed
V_g	Ground speed
δ	Ratio of atmospheric pressure at a given altitude to the atmospheric pressure at sea level
Δ	Change in value of a parameter
%	Percent
μ	Mean value
ρ	Density of air
σ	Track angle
φ	Latitude
$\sum$	Summation of a series of values
ζ	Degrees of freedom of confidence interval
ω_E	Earth’s rotation rate

Appendix 3: Correcting SAR Data to Reference Specifications

A3-1. Corrections to Reference Specifications

This appendix covers corrections that must be made to the tested values of airplane weight, drag, and fuel flow. These corrections address any differences between the conditions at which the airplane was tested and the reference specifications.

For steady-state cruise flight, it is a reasonable approximation to assume lift equals weight and thrust equals drag. Since drag is a function of lift, fuel flow is a function of thrust, SAR is a function of fuel flow, and the FEM is a function of SAR, any deviation from the reference specifications that affect weight, drag, or fuel flow will affect SAR and the FEM. Each of these corrections will result in either increasing or decreasing the test SAR and the resulting FEM.

As stated in Appendix A § A38.5.2.2, each correction method is subject to the approval by the FAA, including any time an applicant considers a particular correction to be unnecessary (with acceptable justification) . Any correction that would result in an increase in SAR (i.e., a lower FEM value) may be considered by the applicant. The effect of not making such a correction would be to increase the FEM value

Table A3-1 lists all the corrections and the conditions under which a given correction might be optional:

Table A3-1. Corrections to reference specifications

Correction	*Paragraph of this appendix*	*Type of correction*	*Conditions under which correction may be optional*	*Remarks*
Latitude effect on g	A3-1.2	Apparent gravity	Test latitude greater than 45.5 degrees	Gravitational acceleration is greatest at the poles and lowest at the equator. Testing at latitudes greater than 45.5 degrees will decrease SAR and increase the FEM as the airplane will be heavier.

Correction	*Paragraph of this appendix*	*Type of correction*	*Conditions under which correction may be optional*	*Remarks*
Altitude effect on g	A3-1.2	Apparent gravity	Test altitude lower than reference altitude	Increasing the height above the Earth's surface reduces the gravitational acceleration. Therefore, testing at an altitude lower than the reference altitude will decreases SAR and increase the FEM due to the effect of altitude on gravitational acceleration.
Centrifugal effect on g	A3-1.2	Apparent gravity	Headwind (negative wind)	The reduction in velocity (relative to the earth) provided by a headwind increases gravitational acceleration due to the centrifugal effect. This increases the airplane weight, which decreases SAR and increases the FEM.
Coriolis effect on g	A3-1.2	Apparent gravity	Test true track angles from 180 to 360 degrees	Flying in a westerly direction, the opposite direction as the Earth's rotation, will increase gravitational acceleration due to the Coriolis effect. This increases the airplane weight which decreases SAR and increases the FEM.
Acceleration/deceleration (energy)	A3-1.3	Drag	(dV_G/dT) greater than 0 (positive acceleration in terms of ground speed)	An accelerated flight condition (excess energy) results in a higher drag force, decreasing SAR and increasing the FEM.

Correction	*Paragraph of this appendix*	*Type of correction*	*Conditions under which correction may be optional*	*Remarks*
Reynolds number	A3-1.4	Drag	Test outside air temperature higher than standard air temperature	Higher temperatures result in higher drag forces due to Reynold number effects. At temperatures higher than the reference temperature, SAR will be decreased and the FEM will increase.
CG position	A3-1.5	Drag	Test CG position forward of the reference CG position	A test CG position forward of the reference CG position results in higher drag, decreasing SAR and increasing the FEM.
Aeroelastics	A3-1.6	Drag	-	Wing aeroelastics may not be a concern depending on the size/weight/payload of the airplane and on wing stiffness.
Fuel lower heating value (LHV)	A3-1.7	Fuel flow	Test fuel LHV less than 43.217 MJ/kg	Testing with fuel that has an LHV lower than the reference value requires more fuel to be burned to achieve the same engine thrust level, hence decreasing SAR and increasing the FEM.
Altitude effect on fuel flow	A3-1.8	Fuel flow	May always be considered optional	Testing at any altitude other than the optimum altitude will result in decreasing SAR and increasing the FEM due to higher fuel burn at off-optimum altitude.
Temperature effect	A3-1.9	Fuel flow	Test outside air	Testing at a temperature

Correction	*Paragraph of this appendix*	*Type of correction*	*Conditions under which correction may be optional*	*Remarks*
on fuel flow			temperature higher than standard day temperature	higher than the reference temperature will result in decreasing SAR and increasing the FEM due to higher fuel burn at higher temperatures.
Engine deterioration level	A3-1.10	Fuel flow	-	In general, this correction should not be made. See paragraph A3-1.10 of this AC.
Electrical and mechanical power extraction and bleed flow	A3-1.11	Fuel flow	Test electrical and mechanical power extraction and bleed flow higher than the reference electrical and mechanical power extraction and bleed flow	Testing with higher than reference power extraction and bleed flow will require more fuel to be burned, hence decreasing SAR and increasing the FEM.
Modifications and non-standard equipment that will not be included on the production airplane	A3-1.12	Drag and/or fuel flow	Any modification or non-standard equipment that reduces SAR	Not accounting for modifications or non-standard equipment that decreases SAR and increases the FEM.

Although the reference specifications include airspeed values that would be selected by the applicant in accordance with §A38.2, there is no correction identified for airspeed. The selection of a reference airspeed is used to determine the target airspeed for acquiring SAR test data for determination of the FEM. For applicants using either SAR data clustered around each of the three reference masses (see § A38.5.3.3) or SAR data obtained over a range of masses (see § A38.5.3.4), no correction is made for any differences between the target reference airspeed and the test airspeed. Since the reference airspeed is generally expected to be the optimum airspeed (see § A38.2), any difference between the test airspeed and the reference airspeed will decrease SAR and increase the FEM. For applicants using first a principles model to show compliance (see paragraph 6.4.4 of this AC), the reference airspeed is used to identify the portion of the performance model to be validated, while the airspeed (or Mach number) at which the SAR value for each reference mass is to be computed from the performance model.

The overall process for determining SAR at the reference specifications for each test data point consists of the following steps:

Step 1) Determine the airplane weight from the airplane mass for both the test condition and the reference specifications for gravitational acceleration, as described in paragraph A3-1.1 below. Determine the drag correction due to the difference in lift between the test weight and the weight at the reference specifications for gravitational acceleration using the airplane's drag model and the relationship that lift equals weight for non-accelerated, level flight, as described in paragraph A3-1.2.1.

Note.— As described in paragraph A3-1.2.2, this step is not necessary when using method 2 for the gravitational acceleration correction.

Step 2) Determine all other applicable drag corrections as described in paragraphs A3-1.3, A3-1.4, A3-1.5, and A3-1.6, and sum them together with the drag correction from paragraph A3-1.2.1.

Step 3) Determine the airplane drag for the test condition. An approximation can be made that for these unaccelerated, level flight test conditions, the airplane drag for the test condition is equal to the engine thrust for the test condition. The engine thrust should be determined from:

1) a calibrated engine or an engine performance model; and

2) the average value of the parameters needed to determine thrust from the engine performance model measured during the test condition.

Step 4) Add the sum of drag corrections from Step 2 to the airplane drag for the test condition from step 3 to obtain the airplane drag corrected to the reference specifications.

Step 5) Determine the change in engine fuel flow for the corrections that were made for drag. The change in engine fuel flow due to the drag corrections can be determined as follows:

$$\Delta \text{Fuel Flow}_{\text{Drag}} = \text{Fuel Flow}_{\text{Ref Drag}} - \text{Fuel Flow}_{\text{Test Drag}}$$

where:

Fuel $\text{Flow}_{\text{Ref Drag}}$ is the fuel flow from an engine performance model at the airplane drag corrected for the reference specifications (assuming thrust equals drag) at the average speed, altitude and temperature for the test condition; and

Fuel $Flow_{Test\ Drag}$ is the fuel flow from an engine performance model at the airplane drag for the test condition (assuming thrust equals drag) at the average speed, altitude and temperature for the test condition.

Step 6) Correct the measured test engine fuel flow to reference specifications as follows:

$$Fuel\ Flow_{ref} = Fuel\ Flow_{test} + \Delta Fuel\ Flow_{Drag} + \Delta Fuel\ Flow_{LHV} + \Delta Fuel\ Flow_{alt} + \Delta Fuel\ Flow_{temp} + \Delta Fuel\ Flow_{Corr\ bleed}$$

where:

Fuel $Flow_{test}$ is the average engine fuel flow measured during the test condition, corrected to the reference engine deterioration level (if applicable) in kg/h;

ΔFuel $Flow_{Drag}$ is the fuel flow correction for drag from Step 5 in kg/h;

ΔFuel $Flow_{LHV}$ is the fuel flow correction for fuel lower heating value calculated from paragraph A3-1.7 in kg/h;

ΔFuel $Flow_{alt}$ is the fuel flow correction for altitude calculated from paragraph A3-1.8 in kg/h;

ΔFuel $Flow_{temp}$ is the fuel flow correction for temperature calculated from paragraph A3-1.9 in kg/h; and

ΔFuel $Flow_{Corr\ bleed}$ is the fuel flow correction for electrical and mechanical power extraction and bleed flow calculated from paragraph A3-1.10 in kg/h.

Step 7) The SAR value corrected to reference specifications is given by the following relationship:

$$SAR_{ref} = \left(\frac{TAS}{Fuel\ Flow_{ref}}\right)$$

where:

SAR_{ref} is the SAR for the reference specifications in km/kg;

TAS is the average airplane true airspeed for the test specifications in km/h; and

Fuel $Flow_{ref}$ is the engine fuel flow for the reference specifications from Step 6 in kg/h.

A3-1.1 Test Weight. Since the weight of the airplane during each test condition cannot be directly measured, it is determined using the following process, in accordance with §§ A38.4.2.1.2 and A38.4.2.3:

Begin with weighing the airplane on the ground, where the local gravitational acceleration can be readily determined using the process described in paragraph A3-1.2.1. The pre-flight airplane mass is determined from the following relationship:

$$Mass_{weighing} = \frac{Weight_{weighing}}{g_{weighing}}$$

where:

$Weight_{weighing}$ is the airplane weight from the scale weighing in N; and

$g_{weighing}$ is the gravitational acceleration at rest at the weighing site latitude and elevation in metres/second^2. See paragraph A-3.1.2.1 for determining $g_{weighing}$ using the weighing site latitude and elevation for the test latitude, φ, and test geometric altitude, h, respectively.

Any change in mass after the weighing, and prior to the test flight, should be accounted for to establish the mass of the airplane at the start of the test flight. The mass for each test condition can be determined by subtracting the integrated fuel flow (mass of fuel burned) from the mass of the airplane at the start of the test flight. The test weight, in N, for each test condition is determined from the following equation:

$$Weight_{test} = (Mass_{test})(g_{test})$$

where:

$Mass_{test}$ is the average mass of the airplane during the test condition in kg; and

g_{test} is the gravitational acceleration at the test latitude, test geometric altitude, test true track, and test ground speed in m/s^2. See paragraph A-3.1.2.1 for determining g_{test}.

A3-1.2 Apparent gravity. Acceleration, caused by the local effects of gravity and inertia, affects the test weight of the airplane. The apparent gravity at the test conditions varies with latitude, altitude, ground speed, and direction of motion, relative to the Earth's axis. The reference gravitational acceleration is the gravitational acceleration, based on g_0, for the airplane travelling in the direction of true north in still air at the reference altitude, a geodetic latitude of 45.5 degrees.

If the test conditions differ from the conditions for the reference gravitational acceleration, the airplane weight at the reference gravitational acceleration will be different than the airplane weight at the test conditions. This will result in a SAR and FEM that are not representative of the SAR and FEM at the reference gravitational acceleration. If the test conditions are at a gravitational acceleration that is less than the reference gravitational acceleration, then a correction will be necessary, as described below.

There are two methods provided below for determining the acceleration correction. Method 1 is a general method for determining the effect of gravitational acceleration on the airplane test weight, which is then used along with the airplane's drag model to determine the effect on airplane drag. This drag correction for gravitational acceleration is then combined with the other drag corrections to determine the effect on airplane fuel flow and the value of SAR.

Method 2 is a simplified approach that does not require use of an airplane drag model. This method is acceptable for FEM certification when SAR is determined by direct flight test measurement of SAR test points in accordance with § A38.1.2.1. Method 2 is based on the same equations for determining g as Method 1, and provides the same results for SAR data at (or near) optimum flight conditions. For SAR data that is not at (or near) reference specifications, Method 2 may result in small errors in the SAR correction for gravitational acceleration correction, but the overall SAR and FEM values will be conservative because these errors will be much smaller than the SAR decrease (thus FEM increase) resulting from testing outside optimum flight conditions.

A3-1.2.1 Method 1 for the gravitational acceleration correction

The overall process for determining the drag correction due to the effect of gravitational acceleration on airplane weight consists of the following steps:

Step 1) Determine the airplane test weight, as described in paragraph A3-1.1.

Step 2) Determine the airplane weight for the reference gravitational acceleration conditions from the airplane test mass and the reference gravitational acceleration:

$$Weight_{ref\ g} = (Mass_{test})(g_{ref})$$

where:

$Weight_{ref\ g}$ is the airplane weight at the reference gravitational acceleration conditions in N;

$Mass_{test}$ is the average mass of the airplane during the test condition in kg;

g_{ref} is the gravitational acceleration for the airplane travelling in the direction of true north in still air at the reference altitude and a geodetic latitude of 45.5 degrees, and based on g_0, in m/s^2; and

g_0 is the standard acceleration due to gravity at sea level and a geodetic latitude of 45.5 degrees, 9.80665 m/s^2.

Step 3) Determine the drag correction for the difference between $Weight_{test}$ and $Weight_{ref\ g}$, as described later in this paragraph.

The value of gravitational acceleration, g, consists of the following components:

a) *Distance from the center of Earth's mass*. This is a function of the geometric altitude and the latitude of the airplane relative to a mathematical representation of the shape of the Earth.

b) *Centrifugal effect*. An airplane in flight will experience a force acting outwards in the radial direction that is proportional to the square of the ground speed of the airplane and inversely proportional to the distance of the airplane from the center of mass of the Earth.

c) *Coriolis effect*. An airplane in flight will experience a force that is proportional to its ground speed, the rotation rate of the Earth, the direction of travel, and the latitude of operation. For example, an airplane flying east, in the same direction that the Earth rotates, will experience a lower gravitational acceleration, while an airplane flying west will experience a higher gravitational acceleration. The effect will be greatest at the equator.

The following equations are based on the World Geodetic System 84 Ellipsoidal Gravity definition. Other formulations and simplifications may provide essentially equivalent results.

The gravitational acceleration experienced for each test data point (g_{test}) is determined as follows:

$$g_{test} = g_{\varphi,alt} + g_{cent} + g_{Coriolis}$$

where:

$g_{\varphi,alt}$ is the component of the gravitational acceleration at zero ground speed, at the test altitude and latitude in m/s^2;

g_{cent} is the component of the gravitational acceleration due to centrifugal effect in m/s^2; and

$g_{Coriolis}$ is the component of the gravitational acceleration due to Coriolis effect in m/s^2.

The gravitational acceleration for the test geometric altitude and latitude component at zero ground speed, $g_{\varphi,alt}$, is determined as follows:

First, determine the component of gravitational acceleration at the test latitude at sea level and zero ground speed from the following equation:

$$g_\varphi = \left(9.7803267714\,\frac{1 + 0.00193185138639\,sin^2\varphi}{\sqrt{1 - 0.00669437999013\,sin^2\varphi}}\right)$$

where φ is the test latitude in degrees.

The component of gravitational acceleration at zero ground speed for the test geometric altitude and latitude is then determined from the following equation:

$$g_{\varphi,alt} = g_{\varphi}\left(\frac{r_e}{r_e + h}\right)^2$$

where:

g_{φ} is the component of gravitational acceleration at the test latitude at sea level and zero ground speed .

h is the test geometric altitude in m; and

r_e is the radius of the Earth at the test latitude, which is determined from the following equation:

$$r_e = \sqrt{\frac{(a^2 cos\ \varphi)^2 + (b^2 sin\ \varphi)^2}{(a\ cos\ \varphi)^2 + (b\ sin\ \varphi)^2}}$$

where:

a is the Earth’s radius at the equator = 6,378,137 m;

b is the Earth’s radius at the pole = 6,356,752 m; and

φ is the test latitude in degrees.

As an alternative to using the equation for $g_{\varphi,alt}$ provided above, $g_{\varphi,alt}$ can be approximated by linear interpolation using Table A3-2:

Table A3-2. $g_{\varphi,alt}$

$g_{\varphi,alt}$											
	Geometric Height (ft)										
Latitude (either North or South) (degrees)	0	5,000	10,000	15,000	20,000	25,000	30,000	35,000	40,000	45,000	50,000
0	9.78033	9.77565	9.77099	9.76632	9.76166	9.75700	9.75234	9.74769	9.74304	9.73840	9.73376
10	9.78188	9.77721	9.77254	9.76787	9.76321	9.75855	9.75389	9.74924	9.74459	9.73994	9.73530
20	9.78637	9.78169	9.77702	9.77235	9.76768	9.76302	9.75836	9.75370	9.74905	9.74440	9.73975
30	9.79325	9.78857	9.76302	9.77921	9.77454	9.76987	9.76521	9.76054	9.75588	9.75123	9.74658
40	9.80170	9.79701	9.79232	9.78764	9.78296	9.77829	9.77362	9.76895	9.76428	9.75962	9.75496
45.5	9.80665	9.80196	9.79727	9.79258	9.78790	9.78322	9.77855	9.77387	9.76920	9.76454	9.75988
50	9.81070	9.80601	9.80132	9.79663	9.79194	9.78726	9.78258	9.77790	9.77323	9.76856	9.76390
60	9.81918	9.81448	9.80978	9.80508	9.80039	9.79570	9.79101	9.78633	9.78165	9.77698	9.77230
70	9.82610	9.82139	9.81668	9.81198	9.80728	9.80259	9.79790	9.79321	9.78853	9.78385	9.77917
80	9.83062	9.82590	9.82120	9.81649	9.81179	9.80709	9.80240	9.79771	9.79302	9.78833	9.78365
90	9.83219	9.82747	9.82276	9.81806	9.81336	9.80866	9.80396	9.79927	9.79458	9.78989	9.78521

The component of the gravitational acceleration due to centrifugal effect, g_{cent}, is determined from the following equation:

$$g_{cent} = -\frac{{V_g}^2}{r_e + h}$$

where:

V_g is the ground speed in m/s;

r_e is the radius of the Earth in m at the test latitude, which is determined from the same equation as provided above for how to determine g_(φ,alt):

$r_e = \sqrt{\frac{(a^2 cos\,\varphi)^2 + (b^2 sin\,\varphi)^2}{(a\,cos\,\varphi)^2 + (b\,sin\,\varphi)^2}}$; and

h is the test geometric altitude in m.

As an alternative to using the equation for g_{ent}, g_{cent} can be approximated by linear interpolation using Table A3-3:

Table A3-3. g_{cent}

g_{cent}									
Ground Speed (kts)	0	100	200	300	400	500	600	700	800
g_{cent}	0.00000	-0.00041	-0.00166	-0.00373	-0.00663	-0.01036	-0.01492	-0.02030	-0.02652

The component of the gravitational acceleration due to Coriolis effect, $g_{Coriolis}$, can be calculated using the following equation:

$$g_{Coriolis} = -2\, \omega_E\, V_g\, cos\, \varphi \sin \sigma$$

where:

ω_E is the Earth's rotation rate = 7.29212 x 10^{-5} radians/s;

V_g is the airplane's ground speed in m/s;

φ is the test latitude in degrees; and

σ is the track angle of the airplane as measured from true north in degrees.

As an alternative to using the equation ~~,~~ $g_{Coriolis}$ can be approximated by linear interpolation using Table A3-4:

Table A3-4. $g_{Coriolis}$

$g_{Coriolis}$ for ground speeds from 200 to 400 knots													
GS (kts)		*200*				*300*				*400*			
Latitude (either N or S) (degrees)		**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**
True Track Angle (degrees)	**0**	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	30	-0.00750	-0.00650	-0.00650	0.00000	-0.01125	-0.00975	-0.00563	0.00000	-0.01501	-0.01300	-0.00750	0.00000
	60	-0.01300	-0.01125	-0.01125	0.00000	-0.01949	-0.01688	-0.00975	0.00000	-0.02599	-0.02251	-0.01300	0.00000
	90	-0.01501	-0.01300	-0.01300	0.00000	-0.02251	-0.01949	-0.01125	0.00000	-0.03001	-0.02599	-0.01501	0.00000
	120	-0.01300	-0.01125	-0.01125	0.00000	-0.01949	-0.01688	-0.00975	0.00000	-0.02599	-0.02251	-0.01300	0.00000
	150	-0.00750	-0.00650	-0.00650	0.00000	-0.01125	-0.00975	-0.00563	0.00000	-0.01501	-0.01300	-0.00750	0.00000

$g_{Coriolis}$ for ground speeds from 200 to 400 knots												
GS (kts)	*200*				*300*				*400*			
Latitude (either N or S) (degrees)	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**
180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
210	0.00750	0.00650	0.00650	0.00000	0.01125	0.00975	0.00563	0.00000	0.01501	0.01300	0.00750	0.00000
240	0.01300	0.01125	0.01125	0.00000	0.01949	0.01688	0.00975	0.00000	0.02599	0.02251	0.01300	0.00000
270	0.01501	0.01300	0.01300	0.00000	0.02251	0.01949	0.01125	0.00000	0.03001	0.02599	0.01501	0.00000
300	0.01300	0.01125	0.01125	0.00000	0.01949	0.01688	0.00975	0.00000	0.02599	0.02251	0.01300	0.00000
330	0.00750	0.00650	0.00650	0.00000	0.01125	0.00975	0.00563	0.00000	0.01501	0.01300	0.00750	0.00000

Table A3-4 (continued)

$g_{Coriolis}$ for ground speeds from 500 to 700 knots													
GS (kts)		*500*				*600*				*700*			
Latitude (either N or S) (degrees)		**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**
True Track Angle (degrees)	**0**	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	30	-0.01876	-0.01624	-0.00938	0.00000	-0.02251	-0.01949	-0.01125	0.00000	-0.02626	-0.02274	-0.01313	0.00000
	60	-0.03249	-0.02814	-0.01624	0.00000	-0.03899	-0.03376	-0.01949	0.00000	-0.04548	-0.03939	-0.02274	0.00000
	90	-0.03751	-0.03249	-0.01876	0.00000	-0.04502	-0.03899	-0.02251	0.00000	-0.05252	-0.04548	-0.02626	0.00000
	120	-0.03249	-0.02814	-0.01624	0.00000	-0.03899	-0.03376	-0.01949	0.00000	-0.04548	-0.03939	-0.02274	0.00000
	150	-0.01876	-0.01624	-0.00938	0.00000	-0.02251	-0.01949	-0.01125	0.00000	-0.02626	-0.02274	-0.01313	0.00000
	180	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	210	0.01876	0.01624	0.00938	0.00000	0.02251	0.01949	0.01125	0.00000	0.02626	0.02274	0.01313	0.00000
	240	0.03249	0.02814	0.01624	0.00000	0.03899	0.03376	0.01949	0.00000	0.04548	0.03939	0.02274	0.00000
	270	0.03751	0.03249	0.01876	0.00000	0.04502	0.03899	0.02251	0.00000	0.05252	0.04548	0.02626	0.00000
	300	0.03249	0.02814	0.01624	0.00000	0.03899	0.03376	0.01949	0.00000	0.04548	0.03939	0.02274	0.00000
	330	0.01876	0.01624	0.00938	0.00000	0.02251	0.01949	0.01125	0.00000	0.02626	0.02274	0.01313	0.00000

The reference gravitational acceleration, g_{ref}, is the gravitational acceleration for the airplane travelling in the direction of true north in still air at the reference altitude and a geodetic latitude of 45.5 degrees. Because the reference gravitational acceleration condition is for the airplane travelling in the direction of true north, the reference gravitational acceleration does not include any Coriolis effect. Because the reference specification is for the airplane travelling in still air, the effect of the centrifugal effect on the reference gravitational acceleration is determined using the airplane's true airspeed (which is the same as the zero wind ground speed). The reference gravitational acceleration can be determined for each test point as follows:

Determine the component of gravitational acceleration for the reference altitude and latitude at zero ground speed using the process defined above, using the reference altitude and 45.5 degrees latitude as the altitude and latitude, respectively, instead of the test values.

Determine the component of the reference gravitational acceleration due to centrifugal effect using the process above, using the airplane's true airspeed as the ground speed.

The reference gravitational acceleration, g_{ref}, is the sum of the component of gravitational acceleration for the reference altitude and latitude at zero ground speed, and the component of the reference gravitational acceleration due to centrifugal effect.

The drag correction for gravitational acceleration (due to the difference between the airplane test weight, $Weight_{test}$, and the airplane weight at the reference gravitational acceleration conditions, $Weight_{ref\ g}$) is determined from the airplane drag model. The airplane's drag model provides the drag coefficient (C_D) as a function of the lift coefficient (C_L) and Mach number. The lift coefficient for $Weight_{test}$ and $Weight_{ref\ g}$ can be determined from the following equation:

$$C_L = \left(\frac{Weight\ /\delta}{70927.5\ M^2 A}\right)$$

where:

C_L is the lift coefficient;

Weight is the weight of the airplane in N (either $Weight_{test}$ or $Weight_{ref\ g}$, depending on which lift coefficient is being calculated);

δ is the ratio of the ambient air pressure at the test altitude to the ambient air pressure at sea level;

M is the airplane's average Mach number during the test condition; and

A is the airplane's reference wing area in m^2.

The drag correction for gravitational acceleration can be determined from the drag equation:

$$\Delta Drag_{grav} = \frac{1}{2}\rho V^2 \left(C_{D_{ref\ g}} - C_{D_{test}}\right)\ A$$

where:

$\Delta Drag_{grav}$ is the drag correction in N for gravitational acceleration;

ρ is the density of air at the test altitude and test temperature in kg/m^3;

V is the airplane's average true airspeed during the test condition in m/s;

A is the airplane's reference wing area in m^2;

$C_{Dref\ g}$ is the drag coefficient from the airplane's drag model at the C_L for $Weight_{ref\ g}$; and

$C_{D\ test}$ is the drag coefficient from the airplane's drag model at the test C_L for $Weight_{test}$.

A3-1.2.2 Method 2 for the gravitational acceleration correction

The process for determining gravitational acceleration correction to SAR consists of the following steps:

Step 1) Determine the airplane test mass, as described in paragraph A3-1.1.

Step 2) Determine g_{test}/g_{ref} for the test latitude by linear interpolation from Table A3-5:

Table A3-5. g_{test}/g_{ref} for latitude (latitude correction)

g_{test}/g_{ref} for latitude (latitude correction)										
Test Latitude (either north or south) (degrees)	0	10	20	30	40	50	60	70	80	90
$(g_{test}/g_{ref})_{lat}$	0.9973	0.9975	0.9979	0.9986	0.9995	1.0004	1.0013	1.0020	1.0024	1.0026

Step 3) Determine g_{test}/g_{ref} to correct for test altitude by linear interpolation from Table A3-6:

Table A3-6. g_{test}/g_{ref} for altitude (altitude correction)

g_{test}/g_{ref} for altitude (altitude correction)											
Reference Altitude - Test Geometric Altitude (ft)	-5000	-4000	-3000	-2000	-1000	0	1000	2000	3000	4000	5000
$(g_{test}/g_{ref})_{alt}$	0.9995	0.9996	0.9997	0.9998	0.9999	1.0000	1.0001	1.0002	1.0003	1.0004	1.0005

Step 4) Determine g_{test}/g_{ref} for headwind/tailwind (centrifugal correction) by linear interpolation from Table A3-7:

Table A3-7. g_{test}/g_{ref} for headwind/tailwind (centrifugal correction)

g_{test}/g_{ref} for headwind/tailwind (centrifugal correction)						
TAS (kts)		200	300	400	500	600
Tailwind (kts)	300	0.9991	0.9989	0.9986	0.9983	0.9981
	200	0.9995	0.9993	0.9991	0.9990	0.9988
	100	0.9998	0.9997	0.9996	0.9995	0.9994
	0	1.0000	1.0000	1.0000	1.0000	1.0000
Headwind (kts)	-100	1.0001	1.0002	1.0003	1.0004	1.0005
	-200	1.0002	1.0003	1.0005	1.0007	1.0009
	-300	1.0001	1.0004	1.0006	1.0009	1.0011

Step 5) Determine g_{test}/g_{ref} for track angle (Coriolis correction) by linear interpolation from Table A3-8:

Table A3-8. g_{test}/g_{ref} for track angle (Coriolis correction)

g_{test}/g_{ref} for track angle (Coriolis correction) for ground speeds from 200 to 400 knots													
GS (kts)		**200**				**300**				**400**			
Latitude (either north or south) (degrees)		**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**
True Track Angle (degrees)	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	30	0.9992	0.9993	0.9996	1.0000	0.9988	0.9990	0.9994	1.0000	0.9985	0.9987	0.9992	1.0000
	60	0.9987	0.9988	0.9993	1.0000	0.9980	0.9983	0.9990	1.0000	0.9973	0.9977	0.9987	1.0000
	90	0.9985	0.9987	0.9992	1.0000	0.9977	0.9980	0.9988	1.0000	0.9969	0.9973	0.9985	1.0000
	120	0.9987	0.9988	0.9993	1.0000	0.9980	0.9983	0.9990	1.0000	0.9973	0.9977	0.9987	1.0000
	150	0.9992	0.9993	0.9996	1.0000	0.9988	0.9990	0.9994	1.0000	0.9985	0.9987	0.9992	1.0000
	180	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	210	1.0008	1.0007	1.0004	1.0000	1.0012	1.0010	1.0006	1.0000	1.0015	1.0013	1.0008	1.0000
	240	1.0013	1.0012	1.0007	1.0000	1.0020	1.0017	1.0010	1.0000	1.0027	1.0023	1.0013	1.0000
	270	1.0015	1.0013	1.0008	1.0000	1.0023	1.0020	1.0012	1.0000	1.0031	1.0027	1.0015	1.0000
	300	1.0013	1.0012	1.0007	1.0000	1.0020	1.0017	1.0010	1.0000	1.0027	1.0023	1.0013	1.0000
	330	1.0008	1.0007	1.0004	1.0000	1.0012	1.0010	1.0006	1.0000	1.0015	1.0013	1.0008	1.0000

Table A3-8 (continued)

g_{test}/g_{ref} for track angle (Coriolis correction) for ground speeds from 500 to 700 knots

GS (kts)	500				600				700			
Latitude (either North or South) (degrees) / **True Track Angle (degrees)**	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**	**0**	**30**	**60**	**90**
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
30	0.9981	0.9983	0.9990	1.0000	0.9977	0.9980	0.9988	1.0000	0.9973	0.9977	0.9987	1.0000
60	0.9967	0.9971	0.9983	1.0000	0.9960	0.9965	0.9980	1.0000	0.9953	0.9960	0.9977	1.0000
90	0.9961	0.9967	0.9981	1.0000	0.9954	0.9960	0.9977	1.0000	0.9946	0.9953	0.9973	1.0000
120	0.9967	0.9971	0.9983	1.0000	0.9960	0.9965	0.9980	1.0000	0.9953	0.9960	0.9977	1.0000
150	0.9981	0.9983	0.9990	1.0000	0.9977	0.9980	0.9988	1.0000	0.9973	0.9977	0.9987	1.0000
180	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
210	1.0019	1.0017	1.0010	1.0000	1.0023	1.0020	1.0012	1.0000	1.0027	1.0023	1.0013	1.0000
240	1.0033	1.0029	1.0017	1.0000	1.0040	1.0035	1.0020	1.0000	1.0047	1.0040	1.0023	1.0000
270	1.0039	1.0033	1.0019	1.0000	1.0046	1.0040	1.0023	1.0000	1.0054	1.0047	1.0027	1.0000
300	1.0033	1.0029	1.0017	1.0000	1.0040	1.0035	1.0020	1.0000	1.0047	1.0040	1.0023	1.0000
330	1.0019	1.0017	1.0010	1.0000	1.0023	1.0020	1.0012	1.0000	1.0027	1.0023	1.0013	1.0000

Step 6) Determine $Mass_{grav}$, as defined below. Couple this mass with the tested SAR values, corrected for reference specifications from Step 7 above, in a SAR versus mass regression model to determine the SAR values at each of the three reference masses of the FEM, and to determine the 90 percent confidence intervals.

$$Mass_{grav} = Mass_{test} \times (g_{test}/g_{ref})_{lat} \times (g_{test}/g_{ref})_{alt} \times (g_{test}/g_{ref})_{cent} \times (g_{test}/g_{ref})_{Coriolis}$$

where:

$Mass_{grav}$ is the mass at which the reference SAR value should be associated in a regression model to provide a SAR versus mass corrected for gravitational acceleration;

$Mass_{test}$ is the test mass, as defined in paragraph A3-1.1, in kg;

$(g_{test}/g_{ref})_{lat}$ is g_{test}/g_{ref} for the test latitude, from Table A3-5;

$(g_{test}/g_{ref})_{alt}$ is g_{test}/g_{ref} for altitude, from Table A3-6;

$(g_{test}/g_{ref})_{cent}$ is g_{test}/g_{ref} for headwind/tailwind (centrifugal correction) from Table A3-7; and

$(g_{test}/g_{ref})_{Coriolis}$ is g_{test}/g_{ref} for track angle (Coriolis correction) from Table A3-8.

A3-1.3 Acceleration/deceleration (energy). Drag determination is based on an assumption of steady, unaccelerated flight. Acceleration or deceleration relative to the ground occurring during a test condition affects the assessed drag level. The reference specification is steady, unaccelerated flight.

The correction for the change in drag force resulting from acceleration during the test condition can be determined from the following equation:

$$\Delta D_{accel} = -Mass_{test}\left(\frac{dV_g}{dT}\right)$$

where:

ΔD_{accel} is the drag correction in N due to acceleration occurring during the test condition;

$Mass_{test}$ is the average mass of the airplane during the test condition, as defined in paragraph A3-1.1 in kg; and

(dV_g/dT) is the change in ground speed over time during the test condition in m/s^2.

A3-1.4 Reynolds number. The RE needs to be calculated to determine its contribution to airplane drag. For a given test condition, the RE is a function of the density and viscosity of air at the test altitude and temperature. The reference RE is derived from the density and viscosity of air from the ICAO standard atmosphere at the reference altitude.

The value of the drag coefficient correction for RE during the test can be expressed as:

$$\Delta C_{D\ RE} = -B\ log\left[\frac{\frac{1}{M}\left(\frac{RE}{m}\right)_{test}}{\frac{1}{M}\left(\frac{RE}{m}\right)_{Ref}}\right]$$

where:

$\Delta C_{D\ RE}$ is the change in drag coefficient when correcting from test RE to the reference RE;

B is a value representing the variation of drag with RE for the specific airplane (see guidance below);

M is Mach number; and

RE/m is RE per m.

One method to obtain B is to use a drag model to obtain the incremental drag variation in response to changing Mach and altitude from a reference cruise condition. The value for B is the value of a single representative slope of a plot of the drag variation, ΔDrag versus $\mathrm{Log}_{10}\left[\frac{1}{M}\left(\frac{RE}{m}\right)x10^{-6}\right]$.

The term $\left[\frac{\frac{1}{M}\left(\frac{RE}{m}\right)_{test}}{\frac{1}{M}\left(\frac{RE}{m}\right)_{Ref}}\right]$ is the term $\frac{1}{M}\left(\frac{RE}{m}\right)$ determined at the temperature and altitude for the test condition, divided by the same term determined at the standard day temperature and the reference altitude for the test mass/δ using the following equation:

$$\frac{1}{M}\frac{RE}{m} = 4.7899\ x\ 10^{5}\ P_S\left(\frac{T_S + 110.4}{T_S^2}\right)$$

where:

RE/m is RE per m;

P_S is static pressure in Pa; and

T_S is static temperature in K.

The effect on airplane drag can then be determined from $\Delta C_{D\ RE}$ and the airplane drag equation as follows:

$$\Delta D_{RE} = \frac{1}{2}\rho\, V^2 \Delta C_{D\ RE}\ A$$

where:

ΔD_{RE} is the airplane drag correction in N due to the test RE being different than the reference RE;

ρ is the density of air at the test altitude and test temperature in kg/m^3;

V is the airplane's average true airspeed during the test condition in m/s;

A is the airplane's reference wing area in m^2; and

$\Delta C_{D\ RE}$ is the change in drag coefficient due to being off the reference RE.

A3-1.5 CG position. The position of the airplane center of gravity (CG) affects the drag due to longitudinal trim.

The drag correction for CG position during the test is the difference between the drag at the reference CG position and the drag at the test CG position. This drag correction can be determined by adjusting the longitudinal trim drag, determined from the airplane's drag model at the reference CG, to adjust for the test CG position. For example, testing at a CG position aft of the reference CG used in the drag model would yield a positive drag correction (that is, a decrease to the test SAR).

A method to determine the change in longitudinal trim drag for CG position from the reference is to first determine the amount of longitudinal trim drag ($C_{D\ Trim\ Ref\ CG}$) that exists at the reference CG position as a function of Mach and C_L. This can be done using wind tunnel testing and analytical methods. The change of the longitudinal trim drag with CG, which is specific to an airplane type design, can be estimated with wind tunnel and analytical methods, and verified by flight test data. This relationship can then be used to determine the longitudinal trim drag ($C_{D\ Trim\ Test\ CG}$) at the test CG position.

Once the trim drag coefficients are determined from the airplane drag model, the airplane drag correction for the CG position can be determined from the drag equation:

$$\Delta D_{Trim\ CG} = \frac{1}{2}\rho\, V^2 \left(C_{D\ Trim\ Ref\ CG} - C_{D\ Trim\ Test\ CG}\right) A$$

where:

$\Delta D_{Trim\ CG}$ is the airplane drag correction in N due to the test CG being different than the reference CG;

ρ is the density of air at the test altitude and test temperature in kg/m^3;

V is the airplane's average true airspeed during the test condition in m/s;

A is the airplane's reference wing area in m^2; and

$C_{D\ Trim\ Ref\ CG}$ and $C_{D\ TrimTest\ CG}$ are the trim drag coefficients from the airplane's drag model at the reference CG and test CG positions, respectively.

A3-1.6 Aeroelastics. Wing aeroelastics may cause a variation in drag as a function of airplane wing mass distribution. Airplane wing mass distribution will be affected by the fuel load distribution in the wings and the presence of any external fuel stores.

There are no simple analytical means to correct for different wing structural loading conditions. If necessary, corrections to the reference specification should be developed by flight test or a suitable analysis process, subject to the approval of the FAA.

The reference specification for the wing structural loading is to be selected by the applicant based on the amount of fuel and/or removable external stores to be carried by the wing, based on the airplane's payload capability and the manufacturer's standard fuel management practices. The reference to the airplane's payload capability is for establishing the zero fuel mass of the airplane, while the reference to the manufacturer's standard fuel management practices is for establishing the distribution of that fuel and how that distribution changes as fuel is burned.

The reference specification for the wing structural loading reference specification should be based on an operationally representative empty weight and payload which defines the zero fuel mass of the airplane. The total amount of fuel loaded for each of the three reference masses would be the reference mass minus the zero fuel mass. Standard fuel management practices will determine the amount of fuel present in each fuel tank. An example of standard fuel management practice is to load the main (wing) fuel tanks before loading the center (body) fuel tanks and to first empty fuel from the center tanks before using the fuel in the main tanks. This helps keep the CG aft and reduces trim drag.

Commercial freighters may be designed from scratch, but more often are derivatives of or are converted from passenger model airplanes. In determining aeroelastic effects, it is reasonable to assume that the reference loading for a freighter is the same as the passenger model it was derived from. If there is no similar passenger model, the reference zero-fuel-mass of a freighter can be based on its payload design density. The payload design density is defined by the full use of the volumetric capacity of the freighter and the highest mass it is designed to carry, expressed in kg/m^3. For example, a typical payload design density for large commercial freighters is 160 kg/m^3.

Using a reference payload significantly lower than the passenger interior limits or structural limited payload could potentially provide a more beneficial aeroelastic effect. An applicant would need to justify the reference payload assumptions in the context of the capability of the airplane and what could be considered typical for the type design.

A3-1.7 Fuel lower heating value. The fuel lower heating value defines the energy content of the fuel. The lower heating value directly affects the fuel flow at a given test condition.

The fuel flow measured during the flight test is corrected to the fuel flow for the reference lower heating value as follows:

$$\Delta Fuel\ Flow_{Corr\ LHV} = Fuel\ Flow_{test\ LHV}\left(\frac{LHV_{test}}{LHV_{Ref}}\right) - Fuel\ Flow_{test\ LHV}$$

where:

ΔFuel $Flow_{Corr\ LHV}$ is the fuel flow correction in kilograms/hour due to the fuel lower heating value being different than the reference fuel lower heating value;

Fuel $Flow_{test\ LHV}$ is the measured fuel flow in kilograms/hour during the test (at the test fuel lower heating value);

LHV_{test} is the fuel lower heating value of the fuel used for the test in MJ/kg; and

LHV_{Ref} is the reference fuel lower heating value = 43.217 MJ/kg.

A3-1.8 Altitude. The altitude at which the airplane is flown affects the fuel flow. As noted in Table A3-1, since testing at any altitude other than the reference optimum altitude increases the FEM, this correction is considered optional. For applicants choosing to make this correction, the selection of the reference altitude depends on the method used for determining SAR for the three reference airplane masses specified in § 38.13 (b). For SAR data clustered around each of the three reference masses (see § A38.5.3.3), a reference altitude is selected by the applicant for each of the specified reference masses. If SAR data is obtained over a range of masses (see § A38.5.3.4), the applicant should select a reference altitude for each test point (since none of the test points are directly associated with any of the reference masses). For applicants using a first principles model (see paragraph 6.4.4 of this AC), the applicant should select a reference altitude for each target Weight/δ value (see Figure 6-6 of this AC).

The engine model should be used to determine the fuel flow at the test altitude (Fuel $Flow_{test\ alt}$) and the fuel flow at the reference altitude (Fuel $Flow_{ref\ alt}$). The fuel flow correction for altitude is determined as follows:

ΔFuel $Flow_{alt}$ = Fuel $Flow_{ref\ alt}$ – Fuel $Flow_{test\ alt}$.

A3.9 Temperature. The ambient temperature affects the fuel flow. The reference temperature is the standard day temperature from the ICAO standard atmosphere at the reference altitude.

The engine model should be used to determine the difference between the fuel flow at the test temperature (Fuel $Flow_{test\ temp}$) and the fuel flow at the reference temperature (Fuel $Flow_{ref\ temp}$). The fuel flow correction for temperature is determined as follows:

ΔFuel $Flow_{temp}$ = Fuel $Flow_{ref\ temp}$ – Fuel $Flow_{test\ temp}$.

A3-1.10 Engine deterioration level. When first used, engines undergo a rapid, initial deterioration in fuel efficiency. Thereafter, the rate of deterioration significantly decreases. Engines with less than the reference deterioration level may be used, subject to the approval of the FAA, as stated in § A38.5.2.2.1.7.1. In such a case, the fuel flow is to be corrected to the reference engine deterioration level using an approved method. Engines with more deterioration than the reference engine deterioration level may be used. In this case, a correction to the reference specification shall not be permitted in accordance with § A38. 5.2.2.1.7.2.

As stated above, a correction should generally not be made for engine deterioration level. If an applicant proposes to use an engine or engines with less than the reference deterioration level for testing, it may be possible to establish a conservative correction level to apply to the test fuel flow to represent engines at the reference deterioration level. Such a correction should be substantiated by engine fuel flow deterioration data from the same engine type or family and approved by the FAA.

A3-1.11 Electrical and mechanical power extraction and bleed flow. Electrical and mechanical power extraction and bleed flow both affect the fuel flow.

The engine model should be used to determine the difference between the fuel flow at the test power extraction ($\text{Fuel Flow}_{\text{test bleed}}$) and bleed flow and the fuel flow at the reference power extraction and bleed flow ($\text{Fuel Flow}_{\text{ref bleed}}$). The fuel flow correction for electrical and mechanical power extraction and bleed flow is determined as follows:

$$\Delta\text{Fuel Flow}_{\text{Corr bleed}} = \text{Fuel Flow}_{\text{ref bleed}} - \text{Fuel Flow}_{\text{test bleed}}.$$

A3-1.12 Modifications and non-standard equipment that will not be included on the production airplane. The test airplane may contain modifications and non-standard equipment, such as flight test instrumentation, that will not be included on the production airplane. Examples of such flight test instrumentation includes trailing cones, precision total air temperature probes, air data booms, telemetry antennas, and engine pressure rakes. Where these modifications or non-standard equipment increase drag or fuel flow, this effect may be corrected out of the tested drag and fuel flow levels used to determine the SAR used for calculating the FEM.

Appendix 4: Confidence Interval Evaluation

A4-1 Direct flight testing

If *n* measurements of SAR ($y_1, y_2, \ldots., y_n$) are obtained under approximately the same conditions, and it can be assumed that they constitute a random sample from a normal population with true population mean, μ, and true standard deviation, σ, then the following statistics can be derived:

$$\bar{y} = estimate\ of\ the\ mean = \frac{1}{n}\left\{\sum_{i=1}^{i=n} y_{(i)}\right\}$$

$$s = estimate\ of\ the\ standard\ deviation\ of\ the\ mean = \sqrt{\frac{\sum_{i=1}^{i=n}(y_i - \bar{y})^2}{n-1}}.$$

From these and the Student t-distribution, the confidence interval, CI, for the estimate of the mean, $\overline{y}$, can be determined as:

$$\mathrm{CI} = \bar{y} \pm t_{\left(1-\frac{\alpha}{2},\zeta\right)} \frac{s}{\sqrt{n}}$$

where $t_{\left(1-\frac{\alpha}{2},\zeta\right)}$ denotes the $\left(1-\frac{\alpha}{2}\right)$ percentile of the single-sided Student t-test with ζ degrees freedom (for a clustered data set $\zeta = n - 1$) and where α is defined such that $100(1-\alpha)$ percent is the desired confidence level for the confidence interval. In other words, it denotes the probability with which the interval will contain the unknown mean, μ. For fuel efficiency metric certification purposes, 90 percent confidence intervals are required and thus $t_{.95,\zeta}$ is used. See Table A4-1 for a listing of values of $t_{.95,\zeta}$ for different values of ζ.

A4-1.1 Regression model

If n measurements of SAR ($y_1, y_2, \ldots, y_n$) are obtained under significantly varying values of mass ($x_1, x_2, \ldots, x_n$), respectively, then a polynomial can be fitted to the data by the method of least squares. For determining the mean SAR, μ , the following polynomial regression model is assumed to apply:

$\mu = B_0 + B_1x + B_2x^2 + \ldots + B_kx^k$.

The estimate of the mean line through the data of the SAR is given by:

$y = b_0 + b_1x + b_2x^2 + \ldots + b_kx^k$.

Each regression coefficient (B_i) is estimated by b_i from the sample data using the method of least squares in a process summarized as follows:

Each observation (x_i,y_i) satisfies the equations:

$y_i = B_0 + B_1x_i + B_2x_i^2 + \ldots + B_kx_i^k + \varepsilon_i$

$= b_0 + b_1x_i + b_2x_i^2 + \ldots + b_kx_i^k + e_i$

where ε_i and e_i are, respectively, the random error and residual associated with the SAR. The random error is assumed to be a random sample from a normal population with mean zero and standard deviation σ. The residual (e_i) is the difference between the measured value and the estimate of the value using the estimates of the regression coefficients and x_i. Its root mean square value (s) is the sample estimate for σ. These equations are often referred to as the normal equations.

Table A4-1. Student t-distribution (for 90 percent confidence) for various degrees of freedom

Degrees of freedom (ζ)	$t_{.95,\ \zeta}$
1	6.314
2	2.920
3	2.353
4	2.132
5	2.015
6	1.943
7	1.895
8	1.860
9	1.833
10	1.812
12	1.782
14	1.761
16	1.746
18	1.734
20	1.725
24	1.711
30	1.697
60	1.671
>60	1.645

The *n* data points of measurements (x_i,y_i) are processed as follows:

Each elemental vector ($\underline{x}_i$) and its transpose ($\underline{x}'_i$) are formed such that:

$\underline{x}_i = (1 \quad x_i \quad x_i^2 \quad . \quad . \quad x_i^k)$, a row vector; and

$$x_i' = \begin{pmatrix} 1 \\ x_i \\ x_i^2 \\ \cdot \\ \cdot \\ x_i^k \end{pmatrix}$$, a column vector.

A matrix $\underline{X}$ is formed from all the elemental vectors $\underline{x}_i$ for i = 1, ….., n. $\underline{X}'$ is the transpose of $\underline{X}$. A matrix $\underline{A}$ is defined such that $\underline{A} = X'\underline{X}$ and a matrix $\underline{A}^{-1}$ is the inverse of $\underline{A}$. In addition, $\underline{y} = (y_1\ y_2\ \ldots\ y_n)$ and $\underline{b} = (b_0\ b_1\ \ldots\ b_2)$, with $\underline{b}$ determined as the solution of the normal equations:

$\underline{y} = \underline{X}\underline{b}$ and $\underline{X}'\underline{y} = \underline{X}'\underline{X}\underline{b} = \underline{A}\underline{b}$

to give

$\underline{b} = \underline{A}^{-1}\underline{X}'\underline{y}$.

The 90 percent confidence interval CI_{90} for the mean value of the SAR estimated with the associated value of the mass x_0 is then defined as:

$CI_{90} = \bar{y}\,(x_0) \pm t_{.95,\zeta}\, s\, v(x_0)$

where $v(x_0) = \sqrt{\underline{x}_0\ \underline{A}^{-1}\ \underline{x}_0'}$.

Thus, $\mathrm{CI}_{90} = \bar{y}\,(x_0) \pm t_{.95,\zeta}\ s\ \sqrt{\underline{x}_0\ \underline{A}^{-1}\ \underline{x}_0'}$,

where:

$\underline{x}_0 = (1\ x_0\ x_0^2 \ldots x_0^k)$;

$\underline{x}'_0$ is the transpose of $\underline{x}_0$;

$\bar{y}\,(x_0)$ is the estimate of the mean value of the SAR at the associated value of the mass x_o;

$t_{.95,\zeta}$ is obtained for ζ degrees of freedom. For the general case of a multiple regression analysis involving K independent variables (i.e. $K + 1$ coefficients), ζ is defined as $\zeta = n - K - 1$ (for the specific case of a polynomial regression analysis, for which k is the order of curve fit, there are k variables independent of the dependent variable, and so $\zeta = n - k - 1$); and

$s = \sqrt{\frac{\sum_{i=1}^{i=n}(y_i - \bar{y}(x_i))^2}{n-K-1}}$ is the estimate of σ, the true standard deviation.

A4-1.2 Worked examples of the determination of 90 percent confidence intervals

Direct flight testing

Example 1: In which the confidence interval is less than the confidence interval limit.

Consider the following set of 6 independent measurements of SAR obtained by flight test around one of the three reference masses of the CO_2 emissions evaluation metric. After correction to reference specifications, the following clustered data set of SAR values is obtained:

Table A4-2. Measurements of SAR — Example 1

Measurement number	*Corrected SAR (km/kg)*
1	0.38152
2	0.38656
3	0.37988
4	0.38011
5	0.38567
6	0.37820

The number of data points (n) = 6

The degrees of freedom (n-1) = 5

The Student t-distribution for 90 percent confidence and 5 degrees of freedom ($t_{(.95\,,5)}$) = 2.015. (See Table A4-1.)

Note.— 6 is the minimum number of test points required, as stated in § A38.5.3.3.

Estimate of the mean SAR ($\overline{SAR}$) for the clustered data set:

$$\overline{SAR} = \frac{1}{n}\left\{\sum_{i=1}^{i=n} SAR_{(i)}\right\} = 0.38282 \text{ km/kg}.$$

Estimate of the standard deviation(s):

$$s = \sqrt{\frac{\sum_{i=1}^{i=n}\left(SAR_{(i)} - \overline{SAR}\right)^2}{n-1}} = 0.00344 \text{ km/kg}\,.$$

Confidence interval determination

The 90 percent confidence interval (CI_{90}) is calculated as follows (see 3.3.2):

$$CI_{90} = \overline{SAR} \pm t_{(.95,n-1)} \frac{s}{\sqrt{n}} = 0.38282 \pm 2.015 \text{ x } \frac{0.00344}{\sqrt{6}} = 0.38282 \pm 0.00283 \text{ km/kg}.$$

Check of confidence interval limits

The confidence interval extends to ±0.00283 km/kg around the mean SAR value of the clustered data set (0.38282 km/kg). This represents ±0.74 percent of the mean SAR value, which is below the confidence interval limit of 1.5 percent, as defined in § A38.5.3.2.

As a result, the SAR value of 0.38282 km/kg associated to one of the reference masses of the FEM can be used for the metric determination.

Example 2: In which the confidence interval exceeds the confidence interval limit.

Consider the following set of 6 independent measurements of SAR obtained by flight test around one of the three reference masses of the FEM. After correction to reference specifications, the following clustered data set of SAR values is obtained:

Table A4-3. Measurements of SAR — Example 2

Measurement number	*Corrected SAR (km/kg)*
1	0.15208
2	0.15795
3	0.15114
4	0.15225
5	0.15697
6	0.15834

The number of data points (n) = 6

The degrees of freedom (n-1) = 5

The Student t-distribution for 90 percent confidence and 5 degrees of freedom ($t_{(.95,5)}$) = 2.015. (See Table A4-1)

Estimate of the mean SAR ($\overline{SAR}$) for the clustered data set:

$$\overline{SAR} = \frac{1}{n}\left\{\sum_{i=1}^{i=n} SAR_{(i)}\right\} = 0.15479 \text{ km/kg.}$$

Estimate of the standard deviation(s):

$$s = \sqrt{\frac{\sum_{i=1}^{i=n}\left(SAR_{(i)} - \overline{SAR}\right)^2}{n-1}} = 0.0033 \text{ km/kg .}$$

Confidence interval determination

The 90 percent confidence interval (CI_{90}) is calculated as follows (see 3.3.2):

$$CI_{90} = \overline{SAR} \pm t_{(.95,n-1)} \frac{s}{\sqrt{n}} = 0.15479 \pm 2.015 \text{ x } \frac{0.0033}{\sqrt{6}} = 0.15479 \pm 0.00271 \text{ km/kg.}$$

Check of confidence interval limits

The confidence interval extends to ±0.00271 km/kg around the mean SAR value of the clustered data set (0.15479 km/kg). This represents ±1.75 percent of the mean SAR value, which is above the confidence interval limit of 1.5 percent, as defined in § A38.5.3.2.

In such case, a penalty equal to the amount that the 90 percent confidence interval exceeds ±1.5 percent is applied to the mean SAR value, i.e. (1.75-1.50) = 0.25 percent. The mean SAR value is therefore be penalized by an amount of 0.25 percent as follows:

$$\overline{SAR} = (1 - \frac{0.25}{100})\ x\ 0.15479 = 0.15440 \text{ km/kg.}$$

As a result, the SAR value of 0.15440 km/kg associated to one of the reference masses of the FEM can be used for the metric determination.

Regression model

Example 3: In which the confidence interval at each of the three reference masses of FEM is less than the confidence interval limit.

Consider the following set of 12 measurements of SAR obtained by flight test at optimum speed and optimum altitude as a function of the airplane gross mass. After SAR correction to reference specifications, the following data set is obtained:

Table A4-4. Measurements of SAR — Example 3

Measurement number and reference mass	*Gross mass (m_i) (kg)*	*Corrected SAR (SAR_i) (km/kg)*
1	17,800	0.928
Low mass (*)	17,825	
2	17,970	0.905
3	18,400	0.908
4	18,850	0.884
5	19,500	0.850
6	19,950	0.845
Mid mass (*)	19,953	
7	20,180	0.833
8	20,350	0.818
9	21,000	0.792
10	21,500	0.781
11	21,870	0.779
High mass (*)	22,080	
12	22,150	0.771

(*) Low, mid and high mass represent the reference masses of the FEM, as defined in §38.13(b)*.*

The number of data points (n) = 12

> *Note.— 12 is the minimum number of test points requested, as stated in* § A38.5.3.4*.*

A representation of the above measurement points is proposed in Figure A4-5.

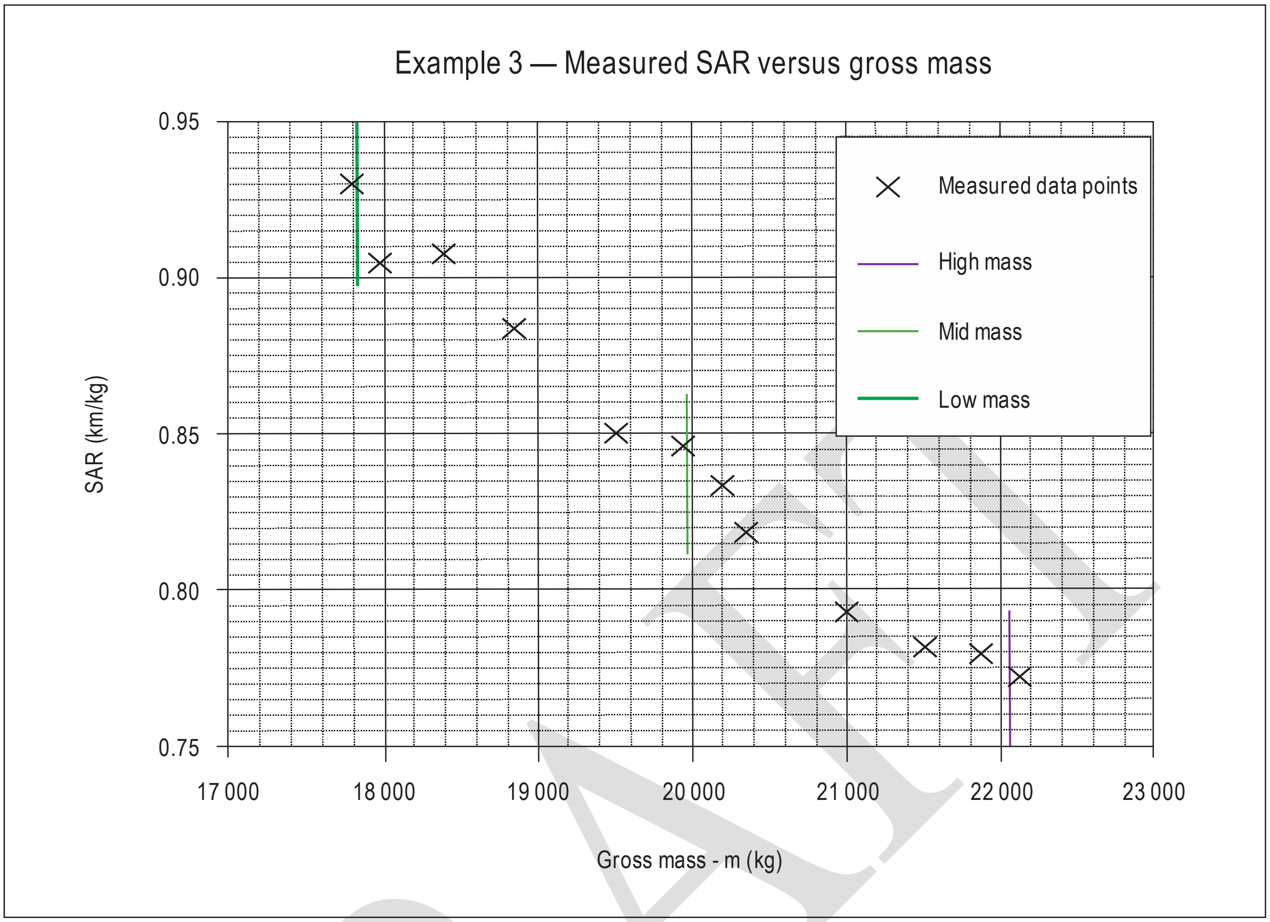


Figure A4-5. Measured SAR versus gross mass — Example 3

Estimate of the mean SAR model by polynomial regression

In order to estimate the SAR model (SAR_{av}) as a function the airplane gross mass (m), a polynomial regression of second order is proposed so that:

$SAR_{av} = B_0 + B_1\, m + B_2\, m^2$

Each observation (m_i, SAR_i), for i = 1, …, 12 satisfies the equation:

$SAR_{(i)} = b_0 + b_1\, m_i + b_2\, m_i^2 + e_i$

where e_i = residual error (difference between the measured SAR value and its estimate).

Under a matrix form this gives:

$$\begin{pmatrix} SAR_1 \\ SAR_2 \\ SAR_3 \\ \vdots \\ SAR_{12} \end{pmatrix} = \begin{pmatrix} 1 & m_1 & m_1^2 \\ 1 & m_2 & m_2^2 \\ 1 & m_3 & m_3^2 \\ \vdots & \vdots & \vdots \\ 1 & m_{12} \cdots & m_{12}^2 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_{12} \end{pmatrix}$$

$\underline{SAR} = \underline{M\ b} + \underline{e}$

where:

$$\underline{SAR} = \begin{pmatrix} 0.928 \\ 0.905 \\ 0.908 \\ 0.884 \\ 0.850 \\ 0.845 \\ 0.833 \\ 0.818 \\ 0.792 \\ 0.781 \\ 0.779 \\ 0.771 \end{pmatrix} \quad \underline{M} = \begin{pmatrix} 1 & 17800 & 17800^2 \\ 1 & 17970 & 17970^2 \\ 1 & 18400 & 18400^2 \\ 1 & 18850 & 18850^2 \\ 1 & 19500 & 19500^2 \\ 1 & 19950 & 19950^2 \\ 1 & 20180 & 20180^2 \\ 1 & 20350 & 20350^2 \\ 1 & 21000 & 21000^2 \\ 1 & 21500 & 21500^2 \\ 1 & 21870 & 21870^2 \\ 1 & 22150 & 22150^2 \end{pmatrix} \quad \underline{b} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} \quad \underline{e} = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_n \end{pmatrix}.$$

The least square principle consists in looking for the parameter values of vector B, minimizing the sum of the squares of residuals, i.e.:

$$\text{Min} \sum_{i=1}^{i=12} e_i^2 = \min \sum_{i=1}^{i=12} (SAR_{(i)} - b_0 - b_1\, m_i - b_2\, m_i^2)^2.$$

It is equivalent to look for the solutions of $\frac{\partial(\sum e_i^2)}{\partial b_j} = 0$ for j = (0, 1, 2).

The solution $\underline{B} = \begin{pmatrix} B_0 \\ B_1 \\ B_2 \end{pmatrix}$ is given by:

$\underline{A^{-1}}\ \underline{M'}\ \underline{SAR}$ (see 3.3.3)

where:

$\underline{M'}$ = transpose of $\underline{M}$; and

$\underline{A^{-1}} = (\underline{M'}\ \underline{M})^{-1}$ = inverse of $(\underline{M'}\ \underline{M})$.

Finally, $\underline{B} = \begin{pmatrix} 2.402921963 \\ -0.000120515 \\ 2.10695\ x10^{-09} \end{pmatrix}$; and

$SAR_{av} = 2.402921963 - 0.000120515\ m + 2.10695.10^{-9}\ m^2$.

Figure A4-6 provides a representation of the mean SAR model as a function of the airplane gross mass.

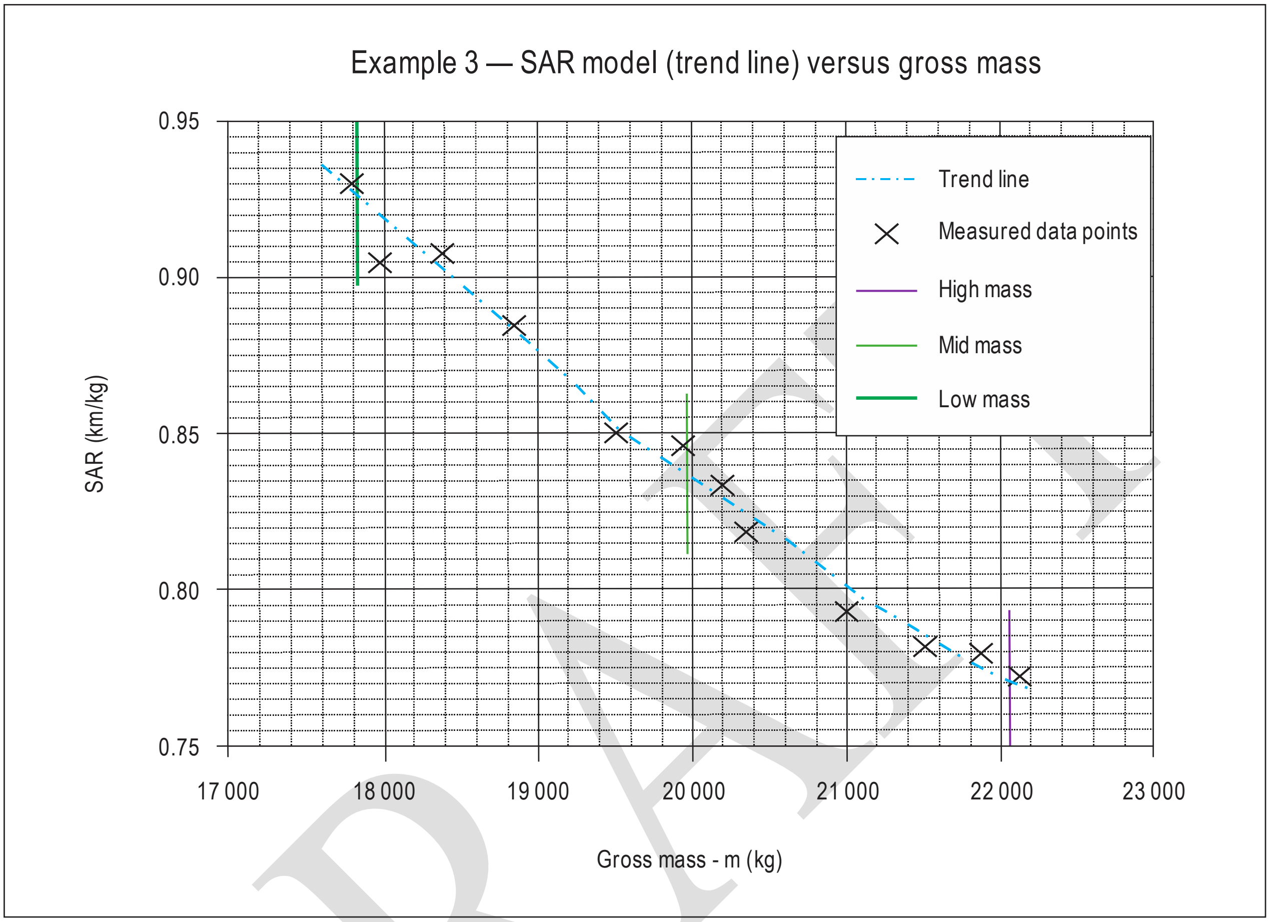


Figure A4-6. SAR model (trend line) versus gross mass — Example 3

The mean SAR values at each of the three reference gross masses of the FEM are as follows:

Table A4-7. Mean SAR values — Example 3

Reference mass	*Mass value (kg)*	*Mean SAR value (km/kg)*
Low mass	17,825	0.92418
Mid mass	19,953	0.83710
High mass	22,080	0.76914

Estimate of the standard deviation(s):

$$s = \sqrt{\frac{\sum_{i=1}^{i=n}(SAR_i - SAR_{av(i)})^2}{n-K-1}} = 0.00765\ km/kg$$

where:

the number of data points (n) = 12;

K = 2 for a second order polynomial regression (see 3.3.3); and

the degrees of freedom (n-K-1) = 9.

Confidence interval determination

The 90 percent confidence interval (CI_{90}) at an airplane gross mass m_0 is calculated as follows (see 3.3.3):

$$CI_{90} = SAR_{av}(m_0) \pm t_{(.95,n-K-1)}\, s \sqrt{\underline{m_0}\ \underline{A^{-1}}\ \underline{m_0'}}$$

where:

the Student t-distribution for 90 percent confidence and 9 degrees of freedom $t_{(.95, 9)}$ = 1.833. (See Table A4-1);

$m_0 = (1\ m_0\ m_0^2)$ and $m_0' = \begin{pmatrix} 1 \\ m_0 \\ m_0^2 \end{pmatrix}$; and

$A^{-1} = (M'\ M)^{-1}$ = inverse of ($M'\ M$).

Figure A4-8 provides a representation of the 90 percent confidence interval as a function of airplane gross mass.

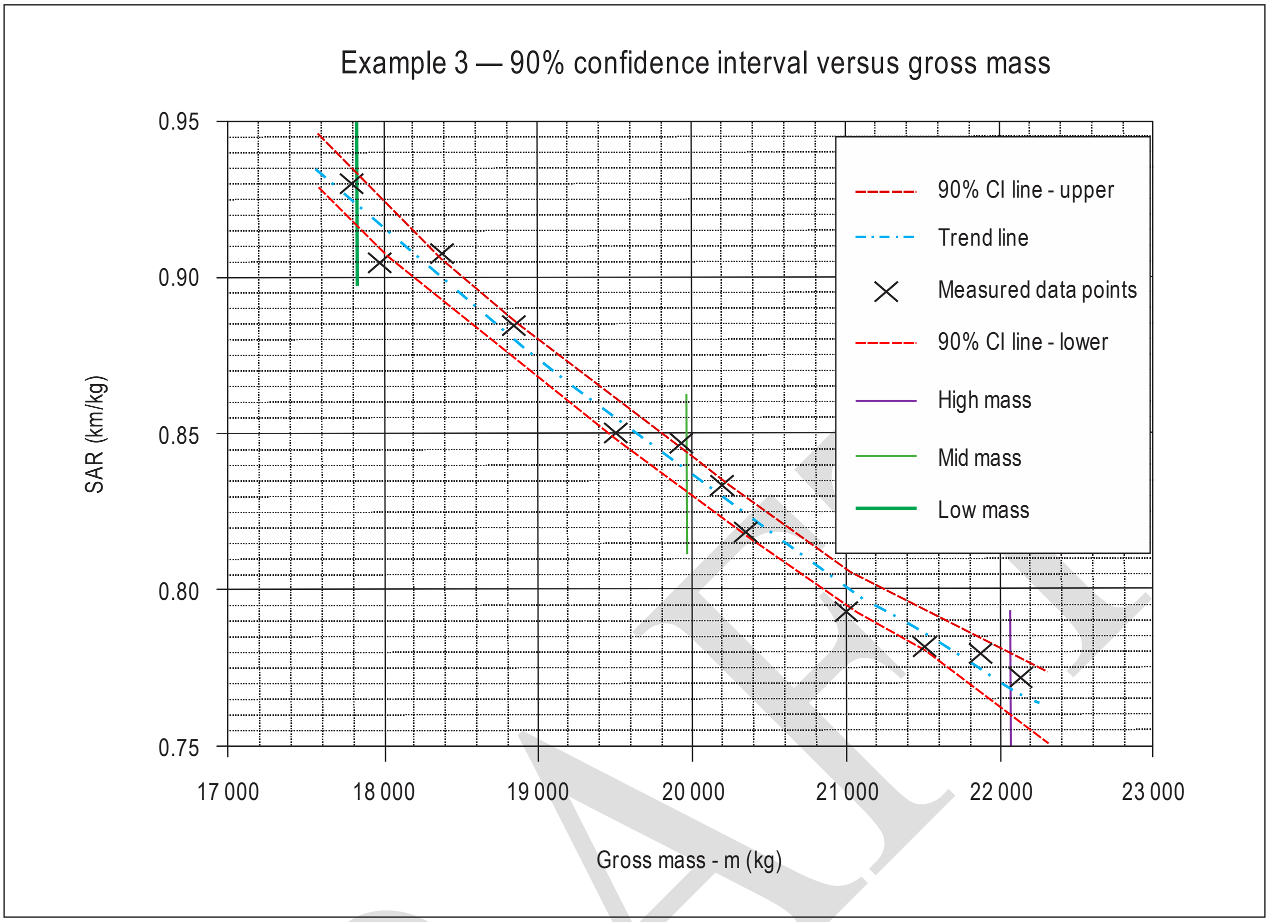


Figure A4-8. 90 percent confidence interval versus gross mass — Example 3

The 90 percent confidence intervals at each of the three reference gross masses of the FEM are as follows:

Table A4-9. Confidence intervals — Example 3

Reference mass	*Mass value (kg)*	*90% confidence interval (kg/km)*
Low mass	17 825	CI_{90} = 0.92418 ± 0.00915
Mid mass	19 953	CI_{90} = 0.83710 ± 0.00619
High mass	22 080	CI_{90} = 0.76914 ± 0.00925

Check of confidence interval limits

For each of the three reference gross masses of the FEM, the confidence interval extends around the mean SAR value by a percentage, as shown in Table A4-10.

Table A4-10. Check of confidence intervals — Example 3

Reference mass	*Mass value (kg)*	*90% confidence interval (percentage of mean SAR)*
Low mass	17,825	(0.00915/0.92418) x 100 = 0.99%
Mid mass	19,953	(0.00619/0.83710) x 100 = 0.74%
High mass	22,080	(0.00925/0.76914) x 100 = 1.2%

The 90 percent confidence intervals at each of the three reference gross masses of the FEM are all below the confidence interval limit of 1.5 percent, as defined in § A38.5.3.2.

As a result, the following mean SAR values associated to each of the three reference masses of the FEM can be used for the metric determination:

Table A4-11. Mean SAR values — Example 3

Reference mass	*Mass value (kg)*	*Mean SAR value (km/kg)*
Low mass	17,825	0.92418
Mid mass	19,953	0.83710
High mass	22,080	0.76914

Example 4: In which the confidence interval of at least one of the three reference masses of the FEM exceeds the confidence interval limit.

Consider the following set of 12 measurements of SAR obtained by flight test at optimum speed and optimum altitude as a function of the airplane gross mass. After SAR correction to reference specifications, the following data set is obtained:

Table A4-12. Measurements of SAR — Example 4

Measurement number and reference mass	*Gross mass (m_i) (kg)*	*Corrected SAR (SAR_i) (km/kg)*
1	17,800	0.932
Low mass (*)	17,825	
2	18,200	0.925
3	18,620	0.913
4	18,890	0.889
5	19,350	0.868
6	19,610	0.848
Mid mass (*)	19,953	
7	19,920	0.838
8	20,510	0.830
9	20,790	0.806
10	21,220	0.815
11	21,480	0.779
High mass (*)	22,080	
12	22,100	0.788

(*) *Low, mid and high mass represent the reference masses of the CO_2 emissions evaluation metric, as defined in* §38 §38.13(b).

The number of data points (n) = 12 (minimum required, as stated in § A38.5.3.4).

A representation of the above measurement points is proposed in Figure A4-13.

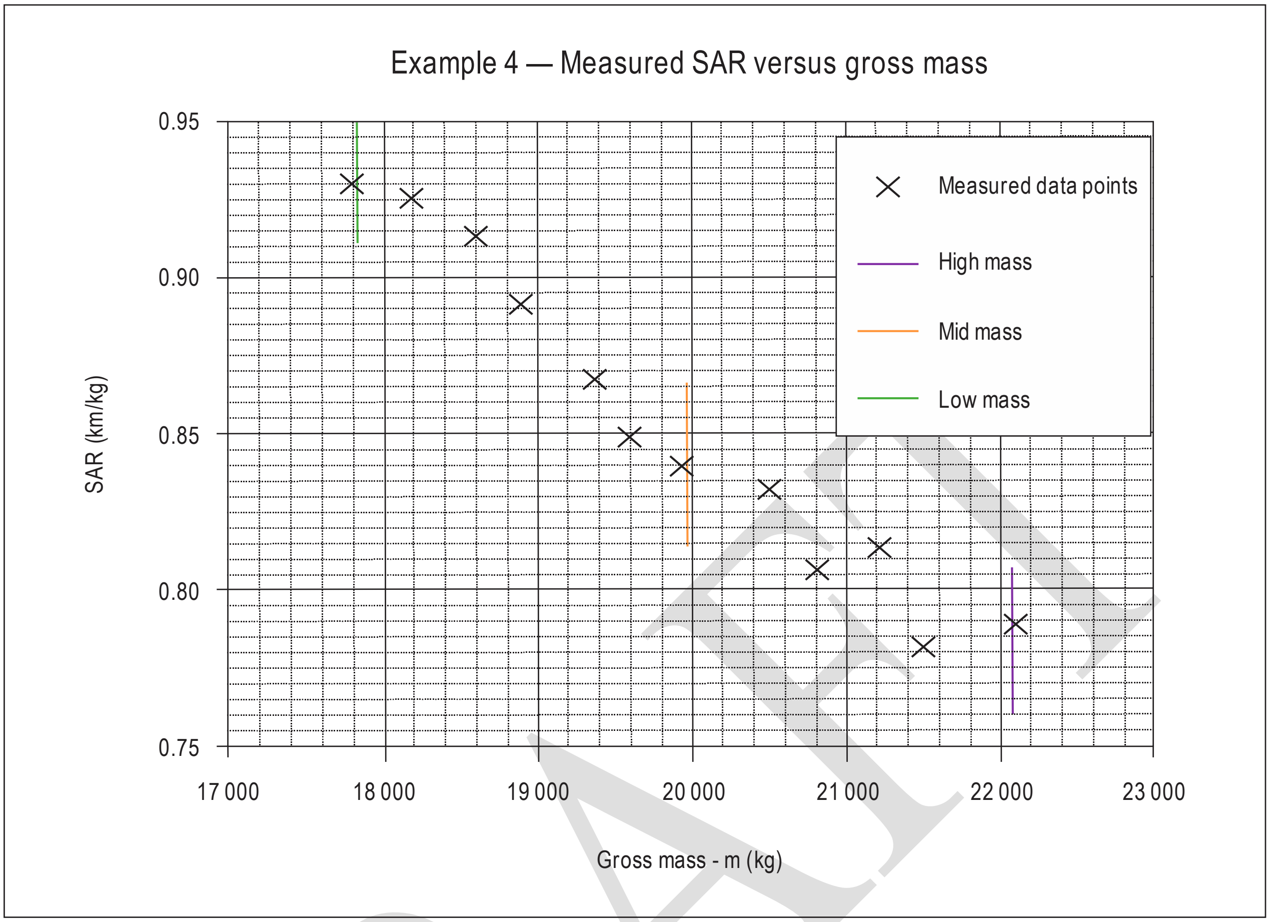


Figure A4-13. Measured SAR versus gross mass — Example 4

Estimate of the mean SAR model by polynomial regression

In order to estimate the SAR model (SAR_{av}) as a function the airplane gross mass (m), a polynomial regression of second order is proposed, so that:

$SAR_{av} = B_0 + B_1\, m + B_2\, m^2$.

Each observation (m_i, SAR_i), for i = 1, …, 12 satisfies the equation:

$SAR_{(i)} = b_0 + b_1\, m_i + b_2\, m_i^2 + e_i$

with e_i = residual error (difference between the measured SAR value and its estimate).

Under a matrix form this gives:

$$\begin{pmatrix} SAR_1 \\ SAR_2 \\ SAR_3 \\ \vdots \\ SAR_{12} \end{pmatrix} = \begin{pmatrix} 1 & m_1 & m_1^2 \\ 1 & m_2 & m_2^2 \\ 1 & m_3 & m_3^2 \\ \vdots & \vdots & \vdots \\ 1 & m_{12} \cdots & m_{12}^2 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_{12} \end{pmatrix}$$

$\underline{SAR} = \underline{M}\ \underline{b} + \underline{e}$

where:

$$SAR = \begin{pmatrix} 0.932 \\ 0.925 \\ 0.913 \\ 0.889 \\ 0.868 \\ 0.848 \\ 0.838 \\ 0.830 \\ 0.806 \\ 0.815 \\ 0.779 \\ 0.788 \end{pmatrix} \quad M = \begin{pmatrix} 1 & 17800 & 17800^2 \\ 1 & 18200 & 18200^2 \\ 1 & 18620 & 18620^2 \\ 1 & 18890 & 18890^2 \\ 1 & 19350 & 19350^2 \\ 1 & 19610 & 19610^2 \\ 1 & 19920 & 19920^2 \\ 1 & 20510 & 20510^2 \\ 1 & 20790 & 20790^2 \\ 1 & 21220 & 21220^2 \\ 1 & 21480 & 21480^2 \\ 1 & 22100 & 22100^2 \end{pmatrix} \quad b = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} \quad e = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_n \end{pmatrix}.$$

The least square principle consists in looking for the parameter values of vector B, minimizing the sum of the squares of residuals, i.e.:

$$\text{Min} \sum_{i=1}^{i=12} e_i^2 = \min \sum_{i=1}^{i=12} (SAR_{(i)} - b_0 - b_1\, m_i - b_2\, m_i^2)^2.$$

It is equivalent to look for the solutions of $\frac{\partial(\sum e_i^2)}{\partial b_j} = 0$ for j = (0, 1, 2).

The solution B = $\begin{pmatrix} B_0 \\ B_1 \\ B_2 \end{pmatrix}$ is given by:

A^{-1} M’ SAR (see 3.3.3)

where:

M’ = transpose of M; and

$A^{-1} = (M'\,M)^{-1}$ = inverse of (M’ M).

Finally, B = $\begin{pmatrix} 3.26727172 \\ -0.000205692 \\ 4.21798\, x10^{-09} \end{pmatrix}$; and

$SAR_{av} = 3.26727172 – 0.000205692\ m + 4.21798.10^{-9}\ m^2$.

Figure A4-14 provides a representation of the mean SAR model as a function of the airplane gross mass.

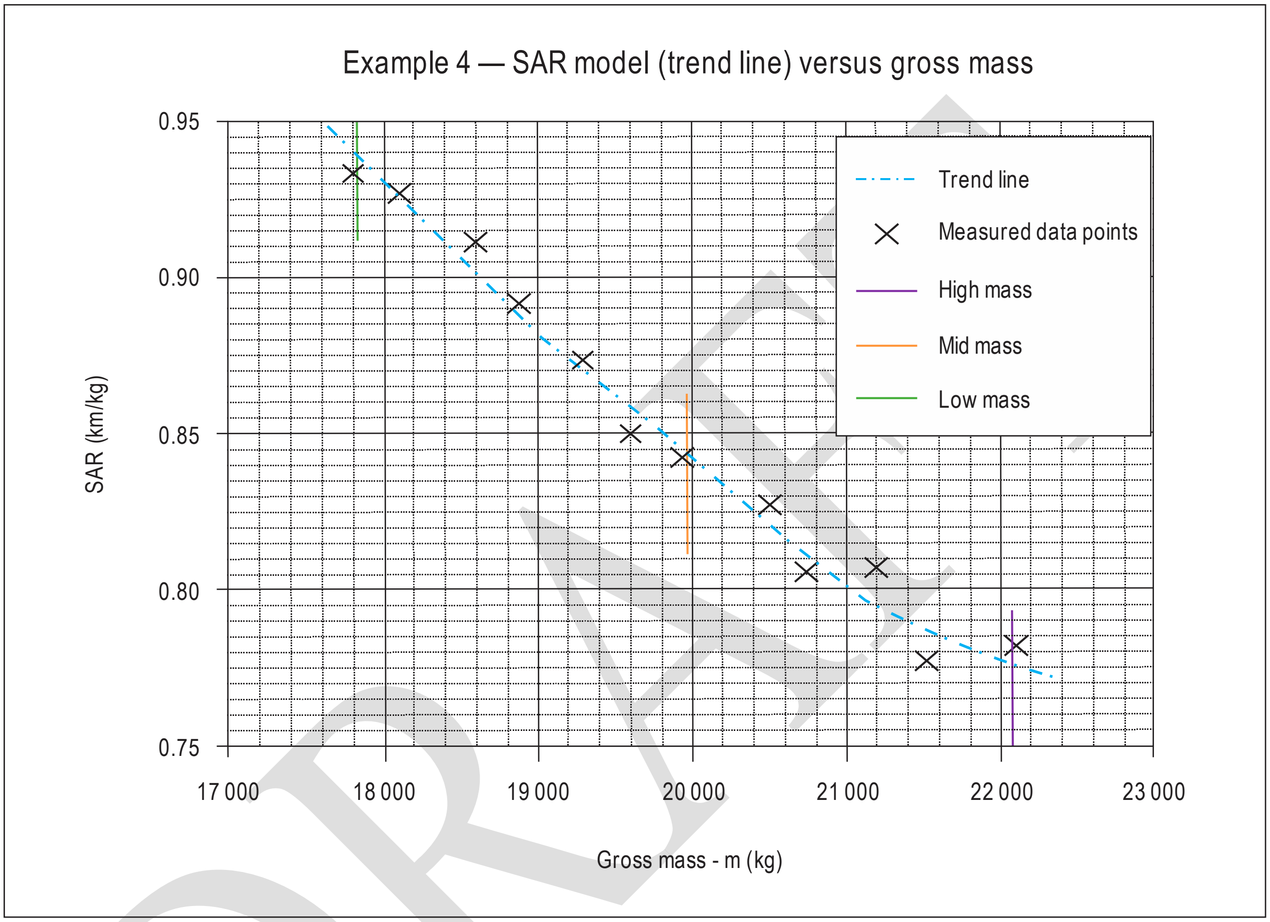


Figure A4-14. SAR model (trend line) versus gross mass — Example 4

The mean SAR values at each of the three reference gross masses of the FEM are as follows:

Table A4-15. Mean SAR value — Example 4

Reference mass	*Mass value (kg)*	*Mean SAR value (km/kg)*
Low mass	17,825	0.94100
Mid mass	19,953	0.84238
High mass	22,080	0.78198

Estimate of the standard deviation(s):

$$s = \sqrt{\frac{\sum_{i=1}^{i=n}(SAR_i - SAR_{av(i)})^2}{n-K-1}} = 0.01050 \text{ km/kg}$$

where:

the number of data points (n) = 12;

K = 2 for a second order polynomial regression (see 3.3.3); and

the degrees of freedom (n-K-1) = 9.

Confidence interval determination

The 90 percent confidence interval (CI_{90}) at an airplane gross mass m_0 is calculated as follows (see 3.3.3):

$$CI_{90} = SAR_{av}(m_0) \pm t_{(.95,n-K-1)}\, s \sqrt{\underline{m_0}\,\underline{A^{-1}}\,\underline{m_0'}}$$

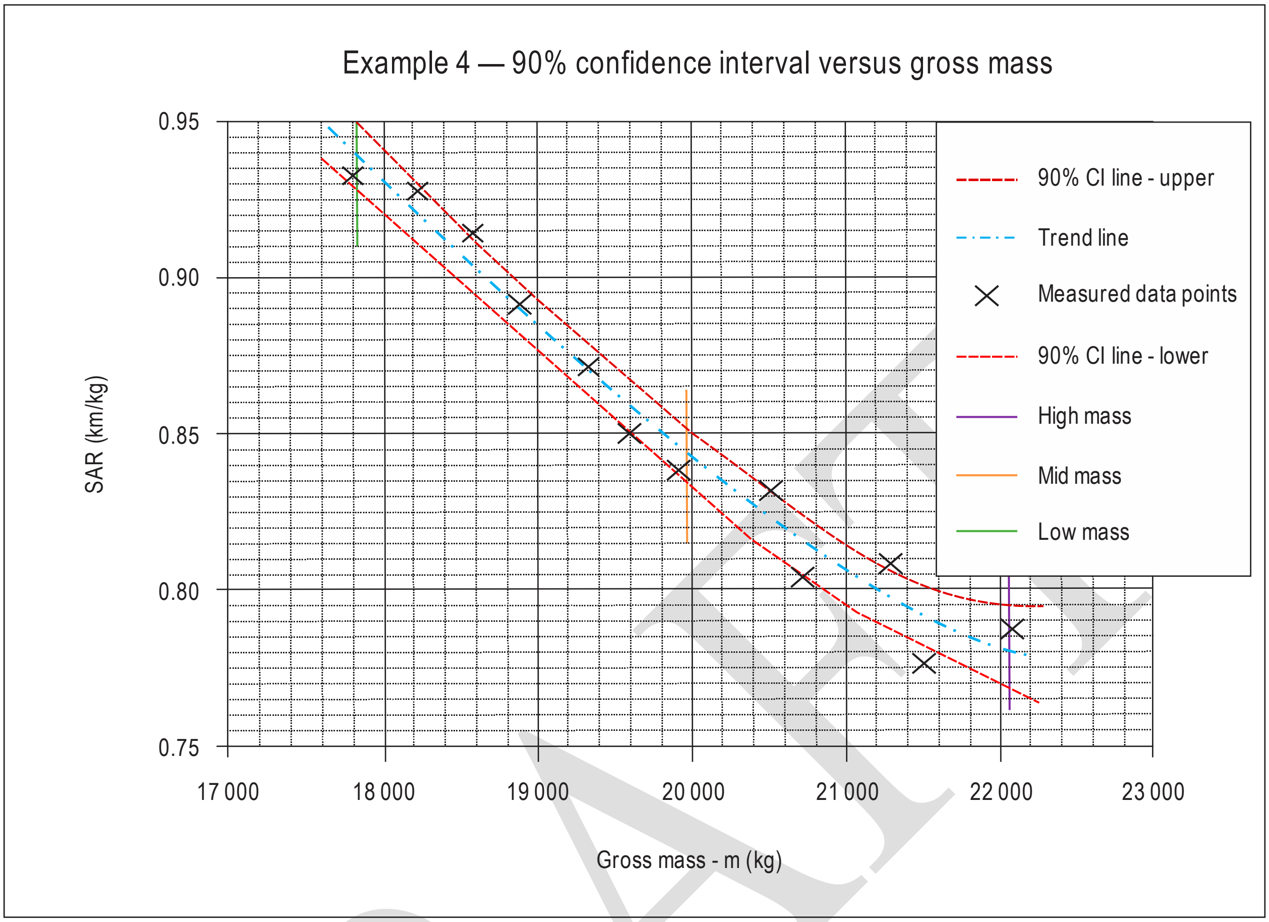
where:

the Student t-distribution for 90 percent confidence and 9 degrees of freedom $t_{(.95,\,9)} = 1.833$. (See Table A4-1);

$m_0 = (1\ m_0\ m_0^2)$ and $m_0' = \begin{pmatrix} 1 \\ m_0 \\ m_0^2 \end{pmatrix}$; and

$A^{-1} = (M'\,M)^{-1}$ = inverse of (M' M).

Figure A4-16 provides a representation of the 90 percent confidence interval as a function of airplane gross mass.



FigureA4-16. 90 percent confidence interval versus gross mass — Example 4

The 90 percent confidence intervals at each of the three reference gross masses of the CO_2 emissions evaluation metric are as follows:

Table A4-17. Confidence intervals — Example 4

Reference mass	*Mass value (kg)*	*90% confidence interval (kg/km)*
Low mass	17,825	CI_{90} = 0.94100 ± 0.01399
Mid mass	19,953	CI_{90} = 0.84238 ± 0.00823
High mass	22,080	CI_{90} = 0.78198 ± 0.01505

Check of confidence interval limits

For each of the three reference gross masses of the FEM, the confidence interval extends around the mean SAR value by an amount provided in Table A4-18.

Table A4-18. Check of confidence intervals — Example 4

Reference mass	*Mass value (kg)*	*90% confidence interval (percentage of mean SAR)*
Low mass	17,825	(0.01399/0.94100) x 100 = 1.52%
Mid mass	19,953	(0.00823/0.84238) x 100 = 0.98%
High mass	22,080	(0.01505/0.78198) x 100 = 1.93%

The 90 percent confidence intervals at the low and high reference gross masses of the FEM are above the confidence interval limit of 1.5 percent, as defined in § A38.5.3.4.

In such case, a penalty equal to the amount that the 90 percent confidence interval exceeds ±1.5 percent is applied to the mean SAR values as follows:

Table A4-19. Corrected SAR values — Example 4

Reference mass	*Mass value (kg)*	*Corrected SAR value (km/kg)*
Low mass	17,825	0.94100 x [1 - (1.52-1.5)/100] = 0.94081
High mass	22,080	0.78198 x [1 - (1.93-1.5)/100] = 0.77862

As a result, the following mean SAR values associated to each of the three reference masses of the FEM can be used for the metric determination.

Table A4-20. Mean SAR values — Example 4

Reference mass	*Mass value (kg)*	*Mean SAR value (km/kg)*
Low mass	17,825	0.94081
Mid mass	19,953	0.84238
High mass	22,080	0.77862

Appendix 5: References

1. International Standards and Recommended Practices, Annex 16 to the Convention on International Civil Aviation, Environmental Protection, Volume III – Aeroplane CO2 Emissions, First Edition, July 2017, Applicable 1 January 2018, Including Amendment 1 of January 1, 2021

2. International Standards and Recommended Practices, Annex 16 to the Convention on International Civil Aviation, Environmental Protection, Doc 9501 Environmental Technical Volume III, Procedures for the CO2 Emissions Certification of Aeroplanes, First Edition, 2018

3. ASTM International D1655-15 entitled "*Standard Specification for Aviation Turbine Fuels*". This ASTM International publication may be obtained from the ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA, 19428-2959 USA, www.astm.org.

4. Defence Standard 91-91, Issue 7, Amendment 3, entitled "*Turbine Fuel, Kerosene Type, Jet A-1*". This United Kingdom Ministry of Defence Standard may be obtained from Defence Equipment and Support, UK Defence Standardization, Kentigern House, 65 Brown Street, Glasgow G2 8EX, UK.

5. ASTM International D4809-13 entitled "*Standard Test Method for Heat of Combustion of Liquid Hydrocarbon Fuels by Bomb Calorimeter (Precision Method)*". This ASTM International publication may be obtained from the ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA, 19428-2959 USA, www.astm.org.

6. ASTM International D4052-11 entitled "*Standard Test Method for Density and Relative Density of Liquids by Digital Density Meter*". This ASTM International publication may be obtained from the ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA, 19428-2959 USA, www.astm.org.

7. ASTM International D445-15 entitled "*Standard Test Method for Kinematic Viscosity of Transparent and Opaque Liquids (and Calculation of Dynamic Viscosity)*". This ASTM International publication may be obtained from the ASTM International, 100 Barr Harbor Drive, PO Box C700, West Conshohocken, PA, 19428-2959 USA, www.astm.org.

DISTRIBUTION:
Manager, ECO Branch, AIR-7E0
Manager, Seattle ACO Branch, AIR-780
Manager, Atlanta ACO Branch, AIR-7A0
Manager, Chicago ACO Branch, AIR-7C0
Manager, Los Angeles ACO Branch, AIR-790
Manager, Engine & Propeller Standards Branch, AIR-6A0

Appendix 6: Advisory Circular 38 Feedback Form

If you find an error in this AC, have recommendations for improving it, or have suggestions for new items/subjects to be added, you may let us know by (1) contacting the Office of Environment and Energy's Emissions Division Manager, AEE-300, at +1-202-267-3566, (2) emailing this form to ralph.iovinelli@faa.gov or (3) Faxing this for to the Attention of the Emissions Division Manager at +1-202-267-5594.

Subject:____________________________ Date: _______________________

Please check all appropriate line items:

☐ An error (procedural or typographical) has been noted on page _______ in paragraph ______.

☐ Recommend paragraph____________on page_______________be changed as follows:

☐ In a future change to this AC, please cover the following subject: *(Briefly describe what you want added.)*

☐ Other comments:

☐ I would like to discuss the above. Please contact me at this email or telephone number:

____

Submitted by: ________________________________ Date: _____